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A PHASE LOCK SYSTEM FOR A SIGNAL MODULATED BY A PN SEQUENCE

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A PHASE LOCK SYSTEM
FOR A SIGNAL MODULATED BY A PN SEQUENCE

by

Yukio Zaima
Lieutenant Commander, Japanese Maritime Self-Defense Force
B.S., U.S. Naval Postgraduate School, Monterey, 1965

Submitted in partial fulfillment
for the degree of

MASTER OF SCIENCE IN ELECTRONICS ENGINEERING

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ABSTRACT

The dynamics of phase locked loops have been studied by many investigators for the cases of either a pure sinusoidal signal or for a pseudo noise (PN) sequence. This dissertation presents an analysis of a phase locked system consisting of a sinusoidal carrier modulated with a PN sequence. The effect of the modulation index of the phase shift modulation and the design of a loop filter for coherent detection of a PN sequence are analysed. Additionally, the error probability for each digit, when a likelihood detector is used in front of the delay lock discriminator for a PN sequence, is also converted to the delay error of the phase lock loop.

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TABLE OF SYMBOLS

$s(t)$ = an m-sequence with the number of digits $M=2^n-1$ in a period, and the duration of a digit d, amplitude ±1

θ = the modulation index of the phase shift modulation

ω_c = the angular frequency of the carrier

A = amplitude of the incoming signal

$\sqrt{2}A\cos(\omega_c t+\theta s(t))$ = incoming signal

A_0 = expected value of the signal amplitude at the demodulator output

$G_s(\omega)$ = the power spectral density of $s(t)$

$n(t)$ = band limited white Gaussian noise

$n_i(t)$ = an in-phase component of $n(t)$ with respect to the carrier

$n_q(t)$ = a quadrature component of $n(t)$ with respect to the carrier

$N(\omega)$ = the Fourier transform of $n(t)$

η = one sided power spectral density of white Gaussian noise (watts/ cycle /second)

$F_1(\omega)$ = the loop filter of the demodulator

$F_2(\omega)$ = the loop filter of the delay lock discriminator

B_0 = band-width parameter of $F_1(\omega)$, (radian/sec)

ω_0 = band-width parameter of $F_2(\omega)$, (radian/sec)

$R_s(\tau)$ = autocorrelation function of $s(t)$

$R_{sn}(\tau)$ = crosscorrelation function of $s(t)$ and $n(t)$

$E[x]$ = the expected value of x

σ_n^2 - the variance of $n(t)$

CHAPTER I

INTRODUCTION

A correlation type detector or a matched filter is known to be optimum for detecting a signal in additive noise under a wide variety of conditions. The noise power is proportional to the effective bandwidth for the case of white Gaussian noise. We can improve the S/N ratio by narrowing the effective bandwidth if it does not restrict the information contained in the signal.

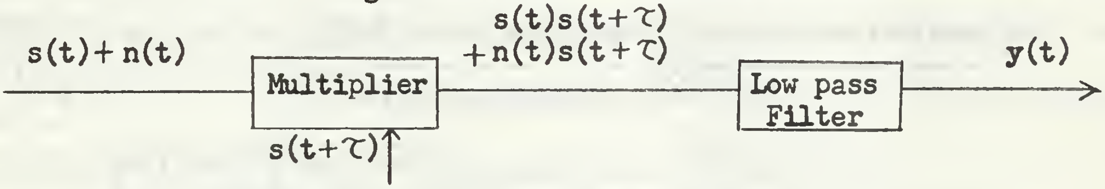


Fig. 1-1 A Correlation Type Detector

Consider the detector in Fig. 1-1. s(t) is the incoming signal and n(t) is additive white Gaussian noise. If we know the signal except for its phase, that is, the phase of the signal is the only information contained in the signal, we can generate $s(t+\tau)$ to multiply with the incoming signal. The low pass filter works like an averager in some sense as described later in section 4-3, and

$$y(t_1,\tau)=\frac{1}{T}\int_{t_1-T}^{t_1}s(t)s(t+\tau)dt+\frac{1}{T}\int_{t_1-T}^{t_1}n(t)s(t+\tau)dt.$$

If s(t) and n(t) are assumed to be stationary, $y(t_1,\tau)$ becomes a function of only τ for long T, and y(t) becomes approximately the sum of the autocorrelation function of s(t) "$R_s(\tau)$" and the crosscorrelation function of s(t) and n(t) "$R_{sn}(\tau)$".

$$y(\tau)\doteq R_s(\tau)+R_{sn}(\tau)$$

If we can find a signal such that its autocorrelation function has a distinguished difference between the values of $\tau=0$ and $\tau\neq 0$, we will be able to utilize it for phase detection.

A sine function $\sqrt{2}$ A cos ω_ct has the autocorrelation function of $R_s(\tau) = A^2 \cos \omega_c \tau$, and the phase locked loop for a sinusoidal function, like that shown in Fig. 1-2, has been studied by many investigators. Among them, R. Jaffe and E. Rechtin showed an optimum filter design for a linearly approximated loop [5], A. J. Viterbi showed the transient response for θ_1 having a second order polynomial in the nonlinear system, [10] W. F. Roland defined the threshold criterion by the minimum of the cycles of slip and showed its utility through experimental results in his thesis for the degree of M. S. [7]

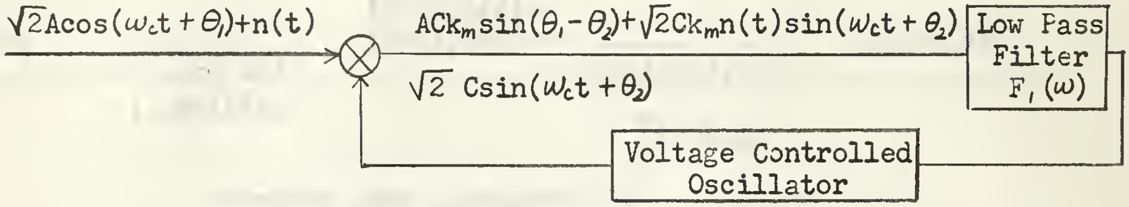

$\sqrt{2}A\cos(\omega_c t+\theta_1)+n(t)$
$ACk_m \sin(\theta_1-\theta_2)+\sqrt{2}Ck_m n(t)\sin(\omega_c t+\theta_2)$
$\sqrt{2}\ C\sin(\omega_c t+\theta_2)$
Low Pass Filter $F_1(\omega)$
Voltage Controlled Oscillator

Fig. 1-2 Phase Locked Loop for Sinusoid

The binary signal has only two possibilities for each digit, ONE or ZERO, or -1 or +1, but sequences of binary digits show many interesting and complex characteristics. Some sequences represent codes for which maximum likelihood detection is possible. They are classified by the crosscorrelation function in each code set; orthogonal, biorthogonal and simplex. [2]

There are also some special group of binary sequences especially adapted for phase detection; the PN sequence, especially the maximum-length linear shift register sequence (m-sequence), will be explained a little more precisely in Chapter 2. R. H. Barker defined "Group Synchronization" so as to pin-point effectively the origin in time for a binary sequence, and showed a matched filter, and also the characteristics and some patterns of this sequence. [1]

The complementary binary sequences studied by S. Jauregui, Jr. used the same kind of matched filters and when used in a differential pair these eliminate small ambiguities in the output. [6] Both sequences can be used in the form of discrete pulse sequences.

PN sequences are adequate for the use of both continuously and periodically incoming signals. J. J. Spilker proposed a delay lock discriminator shown in Fig. 1-3, using the combination of two autocorrelation functions having two digit interval difference τ. [8] J. C. Springett showed a combination of clock and PN sequence to produce a controlling voltage proportional to the phase difference. [9] M. F. Easterling introduced a tracking system using a combination of clock and several codes and produced synchonization, code by code, separately. [3] In this system, only the clock has a phase locked loop. The synchronization of the other codes is evaluated by a correlation indicator. For the system which needs long distance tracking and does not require quick reaction for the variation of the targets dynamics, this will be all right, but if we need automatic synchronization and very rapid acquisition, this will not be adequate.

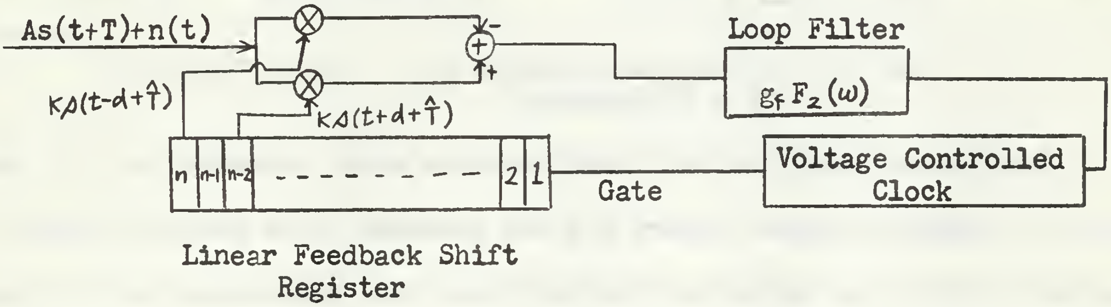


Fig. 1-3 Delay Lock Discriminator for Binary Signal

Since the second one has many weak ambiguous points between distinguished phase lock points, we prefer Spilker's delay lock discriminator rather than Springett's.

The periodic sequence is preferable for phase lock loops to the non-periodic because of the possibility of continuous control.

This paper treats the phase lock system for a carrier phase shift modulated by a PN sequence, as shown in Fig. 1-4. s(t) is a maximum-length linear shift register sequence with the number of digits in a period $M=2^n-1$, and the duration of a digit d. n(t) is white Gaussian noise and has the following narrow band approximation

$$n(t) = n_i(t)\cos\omega_c t + n_q(t)\sin\omega_c t$$

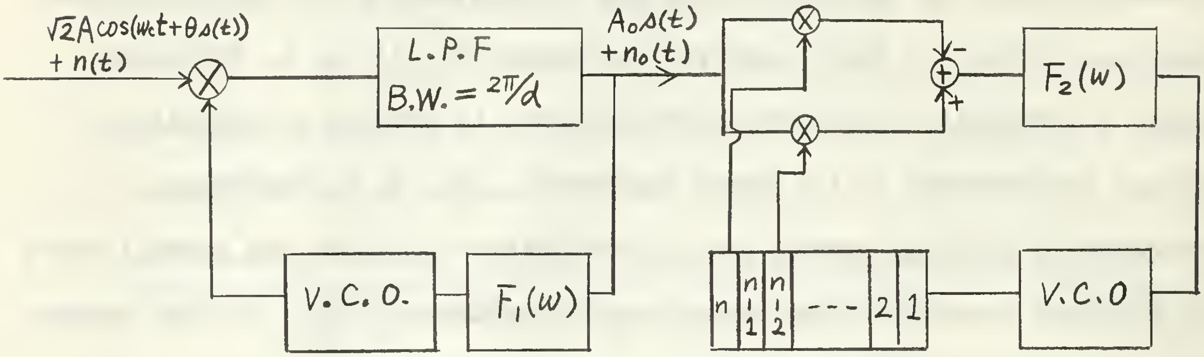


Fig. 1-4 A Phase Lock System for a Signal Modulated by a PN Sequence

Many papers have analyzed each separate stage presented so far, but not the combined system. There are two problems to be solved for this system; what is the effect of θ on the phase lock performance of this system, and if $F_1(\omega)$ and $F_2(\omega)$ have the same type of low pass filter as that derived by Jaffe and Rechtin, how can we determine adequate bandwidths for these filters? θ determines the ratio of the I component and Q component of incoming signals, and also the necessary bandwidth for the input of this system. If θ is large, the output modulating signal will be large, but due to the large phase error of the demodulating stage, the effective amplitude will decrease. The loop bandwidth of each stage is related to the transient response for tracking and the required reduction in the noise output.

This report will summarize those relations determined by other authors for each stage, and will further derive the threshold S/N ratio and the performance of this system. Chapter 2 considers the characteristics of m-sequences. Chapter 3 shows the demodulation of an incoming modulated carrier and the output of this stage.

The delay lock discriminator for an m-sequence will be analyzed in Chapter 4 with the effect of a limiter in front of this stage included in the performance.

Chapter 5 summarises the system results and illustrates by an example the conclusion of the next chapter. This illustration will not show any particularly interesting value for θ or other parameters. It is simply an example of the evaluation of system parameters and of design decision, but it is hoped that this will give a rough idea of the overall performance of such systems and will give some understanding of the role played by each parameter in the design of a system employing PN modulation of a sinusoidal carrier.

CHAPTER II

MAXIMUM-LENGTH LINEAR SHIFT REGISTER SEQUENCE

The autocorrelation function of a binary sequence of length M, $s(i)=\{a_1,a_2,\ldots,a_M\}$, where $a_i=\pm 1$, is defined as

$$\rho_s(j)=\frac{1}{M}\sum_{i=1}^{M}a_i\,a_{i+j} \qquad i+j \text{ : modulo } M$$

If $\rho_s(0)=1$

$$\rho_s(j)=a\neq 1 \qquad (1\leq j<M)$$

then s(i) is called a two-level sequence, and if, moreover, $\rho_s(j)=-\frac{1}{M}$ for $j\neq 0$ (modulo M), the sequence is sometimes called a pseudo-noise (PN) sequence. Exactly, a PN sequence has to have the following three "randomness properties". [4]

R-1 (the balance property) In each period of the sequence the number of ONE's differs from the number of ZERO's by at most 1.

R-2 (the run property) Among the runs of ONE's and of ZERO's in each period, one-half the runs of each kind are of length one, one fourth of each kind are of length two, one eighth are of length three, and so on as these fractions give meaningful numbers of runs.

R-3 (the correlation property) If a period of the sequence is compared, term by term, with any cyclic shift of itself, the number of agreements differs from the number of disagreements by at most 1.

There are four types of PN sequences, corresponding to the length M. [2]

1) $M=2^n-1$	(m-sequences)
2) $M=4t-1$ is prime	(quadratic residue or Legendre sequence)
3) $M=4t-1=4x^2+27$ is prime	(Hall sequence)
4) $M=p(p+2)$ where both p, p+2 are prime	(twin prime sequence)

Among them the maximum-length linear shift register sequence (m-sequence) has the delay-and-add property and is generated by a linear shift register. This is also convenient to generate in the phase lock loop, therefore in this report the m-sequence is used as a PN sequence.

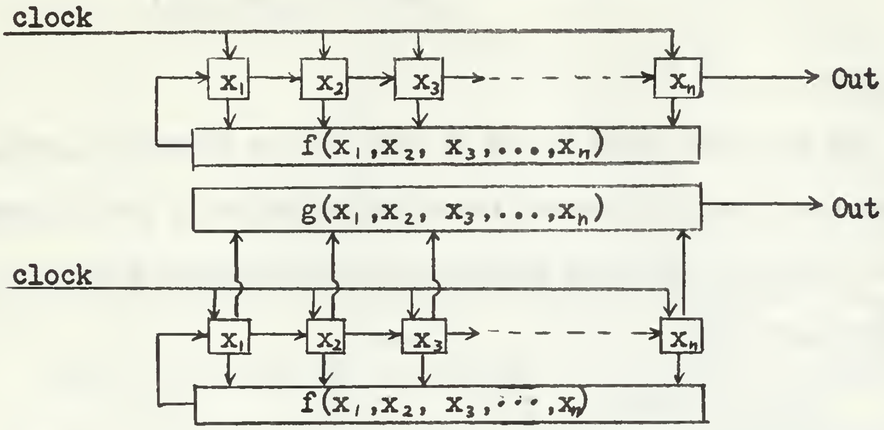

clock
x_1
x_2
x_3
x_n
Out
$f(x_1, x_2, x_3, \ldots, x_n)$
$g(x_1, x_2, x_3, \ldots, x_n)$
Out
clock
x_1
x_2
x_3
x_n
$f(x_1, x_2, x_3, \ldots, x_n)$

Fig. 2-1 n Stage Linear Shift Register

In the n stage linear shift register shown in Fig. 2-1, the output sequence is always periodic with a period $p \leq M = 2^n - 1$, and when period is M, that sequence is called a maximum-length linear shift register sequence. This m-sequence has the delay-and-add property, that is, if the sequence is added term-by-term modulo 2 to a cyclic shift of itself, the resulting sequence is another cyclic shift of the original sequence. This property is possessed only by the binary sequence $\{a_1, a_2, \ldots, a_M\}$ ($a_i = 1$ or 0) which satisfies a linear recursion relation

$$a_k = \sum_{i=1}^{n} c_i a_{k-i}$$

for all k

k-i : (modulo M)

$c_i = 1$ or 0

summation is modulo 2

m-sequence satisfies this.

and $M = 2^n - 1$. [2]

Consider an m-sequence $s(i) = \{a_1, a_2, \ldots, a_M\}$. The autocorrelation function of this sequence $\rho_s(k)$ is

$$\rho_s(k) = \frac{1}{M} \sum_{i=1}^{M} a_i \oplus a_{i+k}$$

$\oplus$, sum of modulo 2

Using the delay-and-add property,

$$\rho_s(k)=\frac{1}{M}\sum_{i=1}^{M}a_{i+j}=\frac{1}{M}\{(\#\text{ of ONE's})\times(-1)+(\#\text{ of ZERO's})\times(1)\}$$

$$=\frac{1}{M}\left\{\frac{M+1}{2}\times(-1)+\frac{M-1}{2}\times(+1)\right\}$$

$$=-\frac{1}{M}\qquad\text{for } k\neq 0\ (\text{modulo } M)$$

$$\rho_s(0)=\frac{1}{M}\sum_{i=1}^{M}a_i\oplus a_i=\frac{1}{M}\sum_{i=1}^{M}1=1$$

(If we replace ONE and ZERO with -1 and +1, we can use multiple instead of $\oplus$). For the wave form s(t) which takes the value of +1 or -1 according to s(i) at time $(i-1)d\leq t<id$, the autocorrelation function $\rho_s(\tau)$ is,

$$\rho_s(\tau)=\frac{1}{T}\int_0^T s(t)s(t+\tau)dt\qquad T=Md$$

For $\tau=0$ or kd (k: integer)

$$\rho_s(0)=\frac{1}{Md}\int_0^{Md}\{s(t)\}^2\,dt=1$$

$$\rho_s(kd)=\frac{1}{Md}\int_0^{Md}s(t)s(t+kd)dt=\frac{1}{Md}\int_0^d dt\sum_{i=1}^{M}a_i a_{i+k}=-\frac{1}{M}$$

If $\tau=kd+\delta$ and $0<\delta<d$, then first $(d-\delta)$ of each digit of s(t) corresponds with the last part $(d-\delta)$ of s(t+kd), and the remaining δ of s(t) corresponds with the first part δ of s(t+(k+1)d).

$$\rho_s(\delta)=\frac{1}{Md}\left\{\int_0^{d-\delta}dt\sum_{i=1}^{M}a_i^2+\int_{d-\delta}^{d}dt\sum_{i=1}^{M}a_i a_{i+1}\right\}$$

$$=1-\frac{\delta}{d}-\frac{\delta}{M}$$

$$\rho_s(kd+\delta)=\frac{1}{Md}\left\{\int_0^{d-\delta}dt\sum_{i=1}^{M}a_i a_{i+k}+\int_{d-\delta}^{d}dt\sum_{i=1}^{M}a_i a_{i+k+1}\right\}$$

$$=-\frac{1}{M}$$

$\rho_s(\tau)$ is an even function and we can get the same result for any negative value of τ, and the autocorrelation function of s(t) has the shape shown in Fig. 2-2.

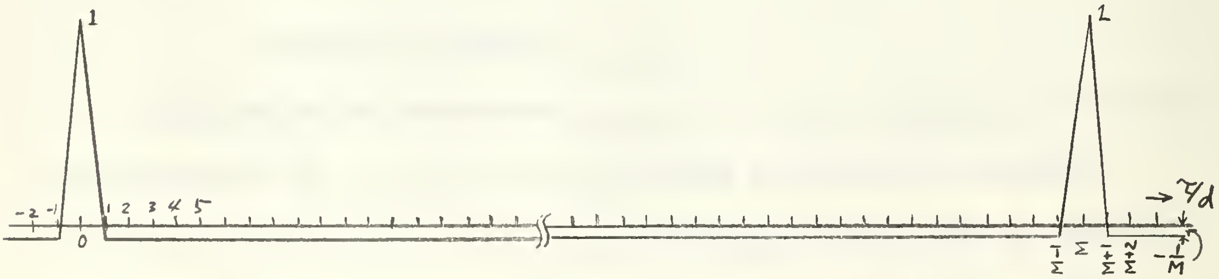


Fig. 2-2 Autocorrelation Function of m-sequence

The power spectral density of s(t), $G_s(\omega)$ is derived by using the Wiener-Khintchine relation.

$$G_s(\omega)=\int_{-\infty}^{\infty}\rho_s(\tau)\, e^{-j\omega\tau}\, d\tau$$
$$=\frac{M+1}{M^2}\left(\frac{\sin(\omega d/2)}{\omega d/2}\right)^2\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\delta\left(\omega-\frac{2\pi n}{Md}\right)+\frac{1}{M^2}\delta(\omega)$$

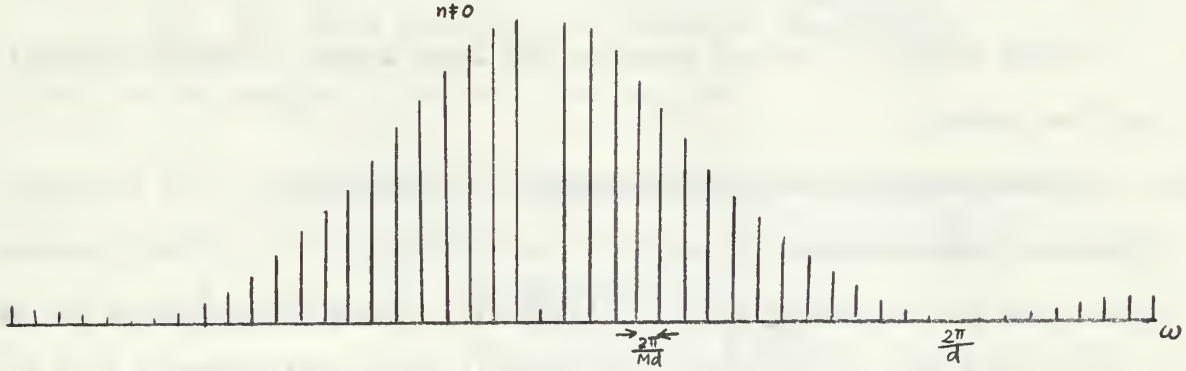


Fig. 2-3 Power Spectral Density $G_s(\omega)$ of m-sequence

When we consider the necessary bandwidth of the band pass filter for the modulated input $\sqrt{2}A\cos(\omega_c t+\theta s(t))$, where $0<\theta<\frac{\pi}{2}$, we can analyze s(t) into frequency components. If we pick up one frequency, then that frequency component will be

$$\sqrt{2}\,\frac{\sqrt{M+1}}{M}\,\frac{\sin\frac{\pi n}{M}}{\frac{\pi n}{M}}\sin\left(\frac{2\pi n}{Md}t+\phi_n\right)$$

Since M is a large number, $\frac{\sqrt{M+1}}{M}\doteq\frac{1}{\sqrt{M}}\ll 1$ and the amplitude of each frequency component is supposed to be very small. A phase modulated carrier ω_c by a modulating frequency ω_m, with small modulation index $\beta\ll 1$, is expressed as

$$\cos(\omega_c t+\beta\sin\omega_m t)\approx\cos\omega_c t-\beta\sin\omega_m t\sin\omega_c t$$

This has frequency components of ω_c, $\omega_c\pm\omega_m$. The frequency components of the signal $\sqrt{2}A\cos(\omega_c t+\theta s(t))$ are derived in the same way as for small modulation index $\beta\ll\frac{\pi}{2}$, therefore in order to preserve almost all power of this incoming signal we can pass only the frequency between $\omega_c\pm\frac{2\pi}{d}$, because almost all the power of s(t) is included inside of $\pm\frac{2\pi}{d}$.

CHAPTER III

PHASE LOCKED LOOP FOR CARRIER

3.1 Coherent Demodulator

Since $s(t) = \pm 1$, we can separate the input signal $\sqrt{2}A\cos(\omega_c t+\theta s(t))$ into two parts.

$$\sqrt{2}A\cos(\omega_c t+\theta s(t))=\sqrt{2}A\cos\theta\ \cos\omega_c t+s(t)\sqrt{2}A\sin\theta\sin\omega_c t$$

If we can lock the phase of the local oscillator to the carrier frequency, we can get the modulating signal $\sqrt{2}A\sin\theta s(t)$. While $\sqrt{2}A\cos\theta\cos\omega_c t$ can be used for that phase lock. Hence, there will be an optimum value of θ to get the maximum signal to noise ratio in the output.

Phase jitter in the local oscillator, caused by the output of $\sqrt{2}As(t)\sin\theta$, plus input noise causes the amplitude variation of signal output in the form of $\sqrt{2}As(t)\sin(\theta+\theta_o)$ and acts to increase the noise input for the next stage.

The crosscorrelation function of $\cos(\omega_c t+\theta s(t))$ and $\cos\omega_c t$ is

$$\begin{aligned}\rho(\tau) &= \frac{1}{2T}\int_{-T}^{T}\cos(\omega_c t+\theta s(t))\cos\omega_c(t+\tau)dt\\ &= \frac{1}{2T}\int_{-T}^{T}\tfrac{1}{2}\big[\cos\theta\cos(2\omega_c t+\omega_c\tau)-s(t)\sin\theta\sin(2\omega_c t+\omega_c\tau)\ \cos\omega_c\tau\cos\theta\\ &\qquad +s(t)\sin\theta\sin\omega_c\tau\big]dt\\ &= \tfrac{1}{2}\cos\theta\cos\omega_c\tau-\tfrac{1}{M}\sin\theta\sin\omega_c\tau\\ &\doteq \tfrac{1}{2}\cos\theta\cos(\omega_c\tau+\phi)\\ \phi &= \tan^{-1}\left(\tfrac{1}{M}\tan\theta\right)\end{aligned}$$

ϕ is negligible when M is large and $\tan\theta$ is not excessive and we can make a phase lock loop like Fig. 3-1.

As far as this phase-lock loop is concerned, the first low pass filter can be neglected and letting the low pass filter be denoted as

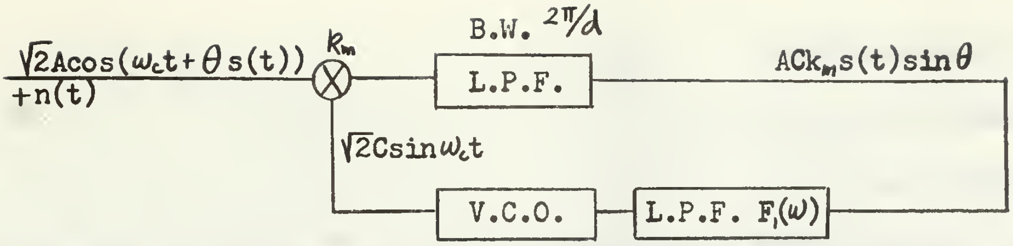


Fig. 3-1 Phase Lock Loop for Coherent Demodulation

$F_1(\omega)$, we can simplify this loop like Fig. 3-2.

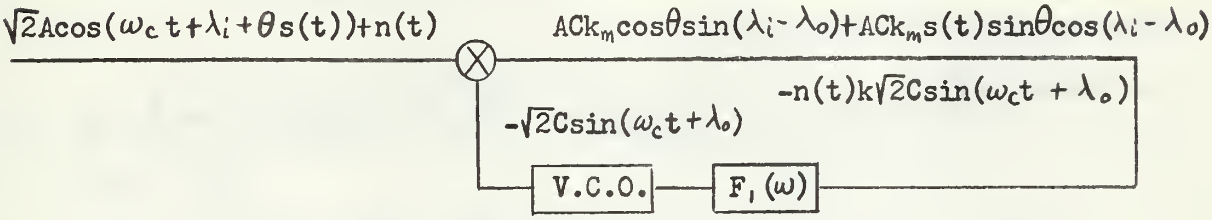

$\sqrt{2}A\cos(\omega_c t+\lambda_i+\theta s(t))+n(t)$
$ACk_m\cos\theta\sin(\lambda_i-\lambda_o)+ACk_m s(t)\sin\theta\cos(\lambda_i-\lambda_o)$
$-n(t)k\sqrt{2}C\sin(\omega_c t+\lambda_o)$
$-\sqrt{2}C\sin(\omega_c t+\lambda_o)$
V.C.O.
$F_1(\omega)$

Fig. 3-2 Phase Lock Loop

Noise input is assumed to be band limited white Gaussian noise, and using narrow band approximation

$$n(t)=n_i(t)\cos\omega_c t+n_q(t)\sin\omega_c t$$

where $n_i(t)$ and $n_q(t)$ are normally distributed time functions varying slowly compared with ω_c, and av.$(n_i)=$ av.$(n_q)=0$, the variances

$\sigma_{n_i}^2=\sigma_{n_q}^2=N=\eta W$

W bandwidth (cycles/sec)

η one sided power spectral density of white Gaussian noise (watts sec/cycle)

When we look at the vector of this input at the vicinity of the phase locked point in Fig. 3-3, $\sqrt{2}A\sin\theta$ and n_q are found as the causes of phase deviation, and since they are independent, their effects will be evaluated in the sum of variances.

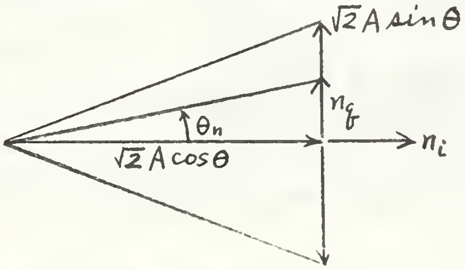


Fig. 3- Vector Diagram for Phase Noise

By approximating

$$\sin(\lambda_i - \lambda_o) = \lambda_i - \lambda_o$$

$$\cos(\lambda_i - \lambda_o) = 1$$

in the vicinity of the phase locked point, we can linearize the loop as shown in Fig. 3-4. The output of the voltage controlled oscillator is the derivative of λ_o and proportional to the input level. Thus we can also modify them in Fig. 3-4.

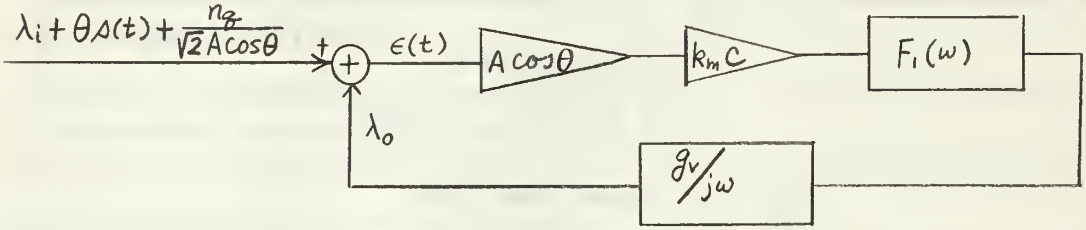


Fig. 3-4 Linearized Phase Lock Loop

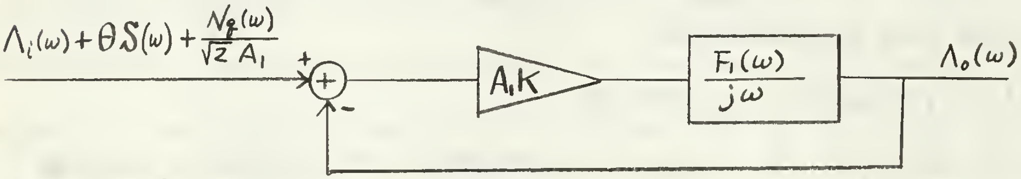

$\Lambda_i(\omega) + \theta S(\omega) + \frac{N_g(\omega)}{\sqrt{2}A_1}$
+
-
A_1K
$\frac{F_1(\omega)}{j\omega}$
$\Lambda_o(\omega)$

Fig. 3-5 Linearized Phase Lock Loop

Let $k_m C g_v = K$

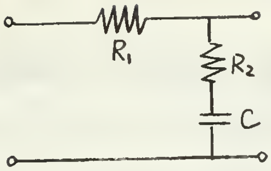
$A\cos\theta = A_1$

then we can get the loop of Fig. 3-5. If we neglect the effect of $\theta S(\omega)$, the solution for optimum performance is shown in the paper of R. Jaffe and E. Rechtin. [5]

Using that result, $F_1(\omega)$ is defined as $F_1(\omega) = \frac{B_o^2 + \sqrt{2}B_o j\omega}{A_1 K j\omega}$ and the closed transfer function $H(\omega)$ is

$$H(\omega) = \frac{A_1 K F_1(\omega)/j\omega}{1 + A_1 K F_1(\omega)/j\omega} = \frac{B_o^2 + \sqrt{2}B_o j\omega}{B_o^2 + \sqrt{2}B_o j\omega + (j\omega)^2}$$

The network for $F_1(\omega)$ is approximately constructed by RC components.



$$C = \frac{\sqrt{2}}{B_o R_2}$$

$$R_1 = R_2\left(\frac{A_o K}{\sqrt{2}\, B_o} - 1\right) \gg R_2$$

From the linearized transfer function $H(\omega)$, we can also derive the error

$$\begin{aligned} \mathcal{E}(\omega) &= \Lambda_i(\omega) - \Lambda_o(\omega) \\ &= \Lambda_i(\omega)\{1 - H(\omega)\} \\ &= \Lambda_i(\omega)\, \frac{-\omega^2}{B_o^2 + \sqrt{2} B_o j\omega - \omega^2} \end{aligned} \qquad (3\text{-}1)$$

For the input of frequency deviation $\Delta\omega$

$$\Lambda_i(\omega) = \frac{\Delta\omega}{-\omega^2}$$

$$\mathcal{E}(\omega) = \frac{\Delta\omega}{B_o^2 + \sqrt{2} B_o j\omega - \omega^2} \qquad (3\text{-}2)$$

$$\begin{aligned} \epsilon(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{E}(\omega) e^{j\omega t}\, d\omega = \frac{\sqrt{2}\Delta\omega}{B_o} e^{-\frac{B_o}{\sqrt{2}}t} \sin\frac{B_o}{\sqrt{2}}t \qquad t > 0 \\ &= 0 \qquad t < 0 \end{aligned} \qquad (3\text{-}3)$$

$$E_T^2 = \int_0^{\infty} \epsilon(t)^2\, dt = \frac{(\Delta\omega)^2}{2\sqrt{2}\, B_o^3} \qquad (3\text{-}4)$$

Max $\epsilon(t)$ is derived by putting $\frac{d\epsilon}{dt} = 0$.

$$t\Big|_{\max\epsilon} = \frac{\pi}{2\sqrt{2}\, B_o} \qquad (3\text{-}5)$$

$$\epsilon(t)\Big|_{\max} = \frac{\Delta\omega}{2.19\, B_o} \qquad (3\text{-}6)$$

The effective bandwidth of $H(\omega)$ is

$$\begin{aligned} \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 d\omega &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1 + 2(\omega/B_o)^2}{1 + (\omega/B_o)^4} d\omega \\ &= \frac{3\sqrt{2}}{4} B_o = 1.06 B_o \qquad \text{(cycles/sec)} \end{aligned} \qquad (3\text{-}7)$$

The phase error due to input noise is normal with zero mean and the variance is

$$\begin{aligned} \sigma_{\theta_n}^2 &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\frac{N_g(\omega)}{\sqrt{2}\, A_i}\right|^2 |H(\omega)|^2\, d\omega \\ &= \frac{\eta}{2 A^2 \cos^2\theta}\, \frac{3\sqrt{2}}{4} B_o = \frac{3\eta B_o}{4\sqrt{2}\, A^2 \cos^2\theta} \end{aligned} \qquad (3\text{-}8)$$

3.2 Noise In the Demodulator Output

As shown in the previous section, the output near the phase locked point is

$$A\cos\theta \sin\theta_o + A s(t) \sin\theta \cos\theta_o + n_g(t) \qquad (3\text{-}9)$$

$$\mathrm{var}(n_\gamma(t)) = \eta\frac{1}{\alpha}$$

If we subtract the required signal $As(t)\sin\theta$, then the remaining is unnecessary and is considered as noise, which is

$$A\cos\theta\sin\theta_0 + As(t)\sin\theta(1-\cos\theta_0) + n_\gamma(t)$$

a) The distribution of θ_0

The phase error θ_0 in eq.(3-9) is the sum of the contribution from the noise input and the self noise of s(t).

Since $H(\omega)$ is assumed to work in the linear region and n_γ is normal, the phase error produced by the noise input θ_n is approximately distributed normally.

The distribution of phase error by the self noise θ_s is derived as follows.

Consider a randomness property of an m-sequence, "R-2" (the Run Property), "Among the runs of ONE's and of ZERO's in each period, the number of runs of length i is $1/2^i$ of the runs of each kind, and $n = \log_2(M+1)$ is the maximum length". When we compare these distributions, we can see each digit has the probability of $\frac{1}{2}$ to be +1 and $\frac{1}{2}$ to be -1. If we take a large number of digits m, then the probability it contains k of +1 is binomial distribution.

$$P(k \text{ of } +1 \text{ out of } m \text{ digits}) = \binom{m}{k}\left(\frac{1}{2}\right)^m$$

Therefore if we take m digits and count the difference of the number of -1 from the number of +1, the probability of the difference to be i is

$$P(\text{difference} = i) = \binom{m}{\frac{m+i}{2}}\left(\frac{1}{2}\right)^m$$

(m and i should be both even, or odd, but since m is large, for a small number of i, we can adjust m as m or m+1 according to the number of i for approximation). For a large number of m, we can approximate it with normal distribution.

Since a low pass filter is, in some sense, an averager, we can conclude that θ_d is also approximately normally distributed with zero mean and with variance equal to $\theta^2 B_0 d$. (See Appendix I).

θ_n and θ_d have different kinds of input and they are independent, therefore, the probability distribution function of $\theta_o = \theta_n + \theta_d$ is also normal with zero mean and variance $\sigma_{\theta_o}^2 = \sigma_{\theta_n}^2 + \sigma_{\theta_d}^2$.

b) The probability distribution of $\sin\theta_o$, and $(1-\cos\theta_o)$ where θ_o has a normal distribution function.

Considering the noise output by θ_o, if we want to get good performance in the next stage, θ_o should be desired to be a very small value in the variance, if not, this system would not work satisfactorily. If θ_o has a small variance, the contribution of θ_o to $A\cos\theta\sin\theta_o$ is approximately evaluated mainly by θ_o neglecting the other terms of series expansion of $\sin\theta_o$, and for $(1-\cos\theta_o)$ by $\theta_o^2/2$.

$$A\cos\theta\sin\theta_o \doteq A\theta_o\cos\theta$$

$$As(t)\sin\theta(1-\cos\theta_o) \doteq \frac{\theta_o^2}{2}As(t)\sin\theta$$

The first one has normal distribution and the second one $\frac{\theta_o^2}{\sigma_{\theta_o}^2}$ is Chi-square distribution with degree of freedom 1.

Looking at θ_o which is the output of $H(\omega)$, we can see that it varies very slowly compared with each digit interval d, but since the ratio of maximum frequency deviation of the input to ω_c is the same as the ratio of the deviation of digit duration to d, and the filter parameter B_0 is wider than the filter parameter ω_0 in the next stage, θ_o varies more rapidly than the fluctuation of phase in the next stage. Therefore the effect of θ_o to the next stage is considered as if $A\cos\theta\sin\theta_o$ were added to n_f as random Gaussian noise in the approximation of $(A\cos\theta)\theta_o$, and since $A\sin\theta(1-\cos\theta_o)$ is approximated by χ_1^2, at the output of the next stage they have the same effect as the decrease of amplitude by a factor like an average of many digits.

The averaged power of $As(t)\sin\theta\cos\theta_0$ is

$$A^2\sin^2\theta\int_{-\infty}^{\infty}\cos^2\theta_0\frac{1}{\sqrt{2\pi}\,\sigma_{\theta_0}}e^{-\theta_0^2/2\sigma_{\theta_0}^2}\,d\theta_0$$

$$=\frac{A^2\sin^2\theta}{2}\left[\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\,\sigma_{\theta_0}}e^{-\theta_0^2/2\sigma_{\theta_0}^2}\,d\theta_0+\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\,\sigma_{\theta_0}}\cos\theta_0\,e^{-\theta_0^2/2\sigma_{\theta_0}^2}\,d\theta_0\right]$$

$$=\frac{A^2\sin^2\theta}{2}\left[1+\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\cos 2\sigma_{\theta_0}x\,e^{-x^2/2}\,dx\right]$$

$$=A^2\sin^2\theta\left[\tfrac{1}{2}+\tfrac{1}{2}e^{-\sigma_{\theta_0}^2/2}\right] \qquad (3\text{-}13)$$

or $$(1-\cos\theta_0)A\sin\theta \doteq \frac{\theta_0^2}{\sigma_{\theta_0}^2}\frac{\sigma_{\theta_0}^2}{2}A\sin\theta$$

$\left(\frac{\theta_0}{\sigma_{\theta_0}}\right)^2$ is distributed in χ_1^2 distribution, and if we take sum of n digits, it becomes χ_n^2

$$E\left(\frac{\chi_n^2}{n}\right)=1,\quad \text{var.}\left(\frac{\chi_n^2}{n}\right)=2/n$$

And as n becomes large, the distribution of the average becomes very close to 1.

$$\text{av.}(A\sin\theta\cos\theta_0)\doteq\left(1-\frac{\sigma_{\theta_0}^2}{2}\right)A\sin\theta \qquad (3\text{-}14)$$

Comparing this with $\sqrt{\frac{1}{2}+\frac{1}{2}e^{-\sigma_{\theta_0}^2/2}}$, for small values of $\sigma_{\theta_0}^2$,

$$\sqrt{\tfrac{1}{2}\left(1+e^{-\sigma_{\theta_0}^2/2}\right)}\doteq\sqrt{1-\frac{\sigma_{\theta_0}^2}{4}}=1-\frac{\sigma_{\theta_0}^2}{2}$$

The result shows good agreement with eq.(3-14).

c) Correlation of θ_0 and $n_q(t)$

From the argument so far, we could derive the approximate output of the discriminator, and it is shown as follows.

$$\sqrt{\tfrac{1}{2}\left(1+e^{-\sigma_{\theta_0}^2/2}\right)}\,(A\sin\theta)s(t) \qquad \text{as the signal}$$

$$+(A\cos\theta)\theta_0+n_q(t) \qquad \text{as noise} \qquad (3\text{-}15)$$

What is the distribution of noise as a whole?

$$\theta_0=\theta_n+\theta_s$$

θ_s is uncorrelated with θ_n and so with $n_q(t)$.

Therefore the correlation of θ_0 and $n_q(t)$ is the correlation of θ_n and $n_q(t)$.

In a linear system of transfer function $H(\omega)$,

$$\theta_n=\frac{N_q(\omega)}{\sqrt{2}\,A\cos\theta}H(\omega)$$

$$h(t)=\sqrt{2}B_0e^{-\frac{B_0}{\sqrt{2}}t}\cos\frac{B_0}{\sqrt{2}}t \qquad \text{for } t>0 \quad (\,h(t)=0 \quad \text{for } t<0)$$

$$\theta_n(t) = \int_0^\infty n(t-\tau)h(\tau)d\tau$$

Correlation of θ_n and $n_g(t)$ is

$$\begin{aligned}
r_{\theta n}(\lambda) &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} n(t+\lambda)\,\theta_n(t)dt \\
&= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} n(t+\lambda)\int_0^\infty n(t-\tau)h(\tau)d\tau dt \\
&= \lim_{T\to\infty}\frac{1}{2T}\int_0^\infty h(\tau)\int_{-T}^{T} n(t+\lambda)n(t-\tau)dtd\tau \\
&= \int_0^\infty h(\tau)\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} n(t+\lambda)n(t-\tau)dtd\tau \\
&= \int_0^\infty h(\tau)R_n(\lambda-\tau)d\tau = \int_0^\infty h(\tau)\eta\,\delta(\lambda-\tau)d\tau \\
&= \frac{B_0\eta}{A\cos\theta}\; e^{-\frac{B_0}{\sqrt{2}}\lambda}\;\cos\frac{B_0}{\sqrt{2}}\lambda
\end{aligned} \qquad (3\text{-}16)$$

for $\lambda = 0$

$$r_{\theta n}(0) = \frac{B_0\eta}{A\cos\theta} \qquad (3\text{-}17)$$

The variances of θ_0 and $n_g(t)$ are

$$\begin{aligned}
\sigma_{\theta_0}^2 &= \frac{3\eta B_0}{4\sqrt{2}\,A^2\cos^2\theta} + \theta^2 B_0 d \\
\sigma_{n_g}^2 &= \eta\frac{1}{d}
\end{aligned} \qquad (3\text{-}18)$$

The normalized correlation coefficient ρ is

$$\rho = \frac{r_{\theta n}(0)}{\sigma_{\theta_0}\,\sigma_{n_g}} \qquad (3\text{-}19)$$

Letting $\theta_0 A\cos\theta = x$

$n_g(t) = y$

$E(x) = E(y) = 0$

$\sigma_x^2 = A^2\cos^2\theta\;\sigma_{\theta_0}^2$

$\sigma_y^2 = \sigma_{n_g}^2$

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\exp\left[\frac{-1}{2(1-\rho^2)}\left(\frac{x^2}{\sigma_x^2} - \frac{2\rho xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2}\right)\right]$$

Changing variables $x+y=u$

$y = v$

$$\begin{aligned}
p(u) &= \int_{-\infty}^{\infty}\frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\exp\left[\frac{1}{2(1-\rho^2)}\left(\frac{(u-v)^2}{\sigma_x^2} - \frac{2\rho(u-v)v}{\sigma_x\sigma_y} + \frac{v^2}{\sigma_y^2}\right)\right]dv \\
&= \exp\left[-\frac{u^2}{2(\sigma_x^2+2\rho\sigma_x\sigma_y+\sigma_y^2)}\right]\int_{-\infty}^{\infty}\frac{1}{2\pi\sqrt{1-\rho^2}\sqrt{\sigma_x^2+2\rho\sigma_x\sigma_y+\sigma_y^2}}\,e^{-\frac{z^2}{2(1-\rho^2)}}dz
\end{aligned}$$

where $z = \sqrt{\frac{1}{\sigma_x^2}+\frac{2\rho}{\sigma_x\sigma_y}+\frac{1}{\sigma_y^2}}\;v + \frac{u}{\sigma_x}\,\frac{(1/\sigma_x + \rho/\sigma_y)}{\sqrt{\frac{1}{\sigma_x^2}+\frac{2\rho}{\sigma_x\sigma_y}+\frac{1}{\sigma_y^2}}}$

$$p(u) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_x^2+2\rho\sigma_x\sigma_y+\sigma_y^2}}\exp\left[-\frac{u^2}{2(\sigma_x^2+2\rho\sigma_x\sigma_y+\sigma_y^2)}\right]$$

$$\sigma_n^2 = \sigma_x^2 + 2\rho\sigma_x\sigma_y + \sigma_y^2$$

$$u = n_0(t)$$

The output is now simplified as

$$\sqrt{\tfrac{1}{2}(1+e^{-\sigma_{\theta_0}^2/2})}\,(A\sin\theta)s(t) + n_0(t)$$

$n_0(t)$ is normal with zero mean and variance σ_n^2.

The power spectral density of this noise is not flat in the bandwidth of 1/d, because θ_0 has the approximate bandwidth of B_0. Let the power spectral density of $n_0(t)$ be $G_n(\omega)$

$$\sigma_n^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_n(\omega)\,d\omega$$

If we divide this by $G_n(\omega)\big|_{\omega=0}$, then we can get the effective bandwidth B_ℓ. The low pass filter used in the next stage has narrower bandwidth than B_0, then we can use $(\frac{\sigma_n^2}{B_\ell})$ as the approximation of $G_n(\omega)$ because the value of $G_n(\omega)$ inside of that bandwidth ω_0 mainly contributes to the output.

Let the Fourier transform of $n_q(t)$ be $N_q(\omega)$, then the Fourier transform of $(A\cos\theta)\theta_0$ is $\frac{N_q(\omega)}{\sqrt{2}}H(\omega) + (A\cos\theta)\,\textcircled{H}_s(\omega)$

$$\frac{N_q(\omega)}{\sqrt{2}}H(\omega)\Big|_{\omega=0} = \frac{N_q(0)}{\sqrt{2}}$$

$$(A\cos\theta)\textcircled{H}_s(\omega)\Big|_{\omega=0} = (A\cos\theta)\frac{1}{M} \doteq 0$$

$$G_n(\omega)\Big|_{\omega=0} = \left\{\left|N_q(\omega) + \frac{N_q(\omega)}{\sqrt{2}}H(\omega)\right|^2\right\}_{\omega=0} \doteq 2.91\eta$$

CHAPTER IV

DELAY LOCK DISCRIMINATOR FOR A BINARY SEQUENCE

4.1 Tracking and Acquisition Performance of a Delay-lock Discriminator

This has been solved by J. J. Spilker, [8] and here we would like to follow the general results. This section explains the tracking and acquisition performance, and the next section shows the effect of noise on the performance.

If we take the difference of the outputs from two crosscorrelators which differ two digit intervals in Fig. 4-1, and separate it into an averaged value $A_0kD_d(\epsilon)$ and another fluctuating part (self-noise) $A_0kn_\Delta(t,\epsilon)$, these are derived as follows.

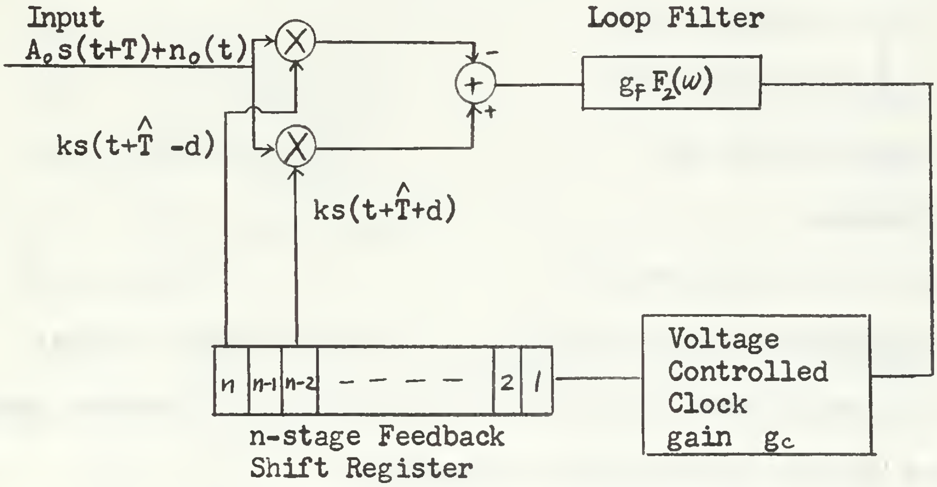
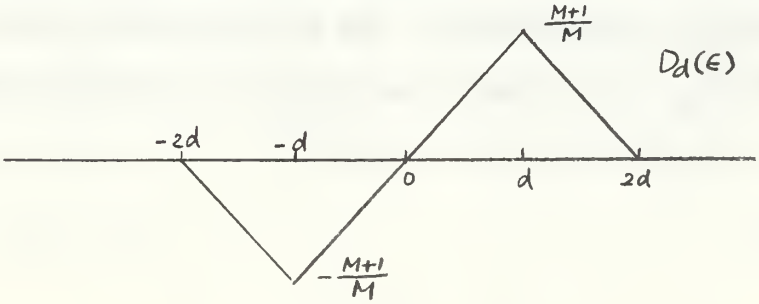


Fig. 4-1 Delay Lock Discriminator

The autocorrelation function of an m-sequence is already shown in Fig. 2-1, and from that figure we can draw $D_d(\epsilon)$ as this.



$$D_d(\epsilon) = E\big[s(t+T)s(t+d+\hat{T})-s(t+T)s(t-d+\hat{T})$$

$$= \{R_\Delta(\epsilon-d)-R_\Delta(\epsilon+d)\} \qquad (4\text{-}1)$$

The self-noise in the output of the multiplier network is

$$n_\Delta(t,\epsilon) = s(t+d+\hat{T})s(t+T)-s(t-d+\hat{T})s(t+T)-D_d(\epsilon) \qquad (4\text{-}2)$$

For $\epsilon=kd$, k: integer, using the delay-and-add property,

a) $k=0$, (modulo M)

$$\begin{aligned} n_\Delta(t,kd) &= s(t+d+T)s(t+T) - s(t-d+T)s(t+T) \\ &= s(t+T+d)s(t+T)-s(t+T-d+d)s(t+T-d) \\ &= s(t+T+jd)-s(t+T-d+jd) \\ &= s(t+T+jd)-s(t+T+(j-1)d) \end{aligned} \qquad (4\text{-}3)$$

b) $k=\pm 1$, (modulo M)

$$\begin{aligned} n_\Delta(t,kd) &= s(t+d+T\mp d)s(t+T)-s(t-d+T\mp d)s(t+T)-D_d(\pm d) \\ &= \begin{cases} 1-s(t+T+id)-\frac{M+1}{M} \\ s(t+T+id)-1+\frac{M+1}{M} \end{cases} \\ &= \mp\left[s(t+T+id)-\frac{1}{M}\right] \end{aligned} \qquad (4\text{-}4)$$

c) $k=$ other integer

$$n_\Delta(t,kd) = s(t+T+rd)-s(t+T+qd) \qquad (4\text{-}5)$$

Further discussions of this self-noise, i.e. its power spectral density and the output from the loop will be explained in the next section.

A) The output of the crosscorrelator network is

$$kA_0\left[D_d(\epsilon)+n_\Delta(t,\epsilon)+n_0(t)/A_0\right]$$

This is filtered by a low pass filter $g_f F_2(\omega)$ and drives the voltage controlled clock generator. This relation is now shown in the following Fig. 4-2. Obviously the control part of the crosscorrelator output $D_d(\epsilon)$ is a function of $T-\hat{T}=\epsilon$, and T is the integral of the controlled frequency.

$$\hat{T} = \frac{d}{j\omega} kA_0 g_c g_f F_2(\omega)\left[D_d(\epsilon)+n_\Delta(\omega,\epsilon)+n_0(\omega)/A_0\right]$$

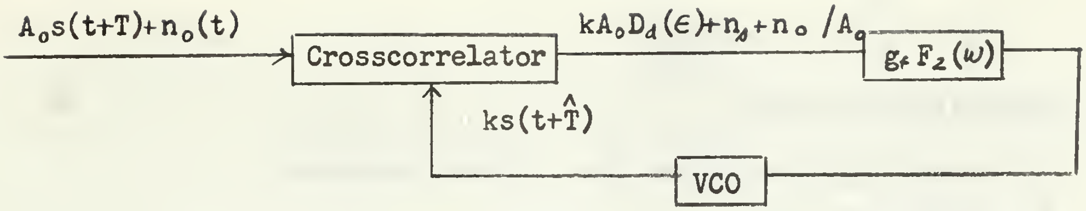


Fig. 4-2 Simplified Delay Lock Discriminator

In the locked-on state, $\epsilon<d$,

$$D_d(\epsilon)=\frac{M+1}{M}\frac{\epsilon}{d}$$

$$\epsilon=T-\hat{T}$$

$$j\omega\hat{T}=dkA_o g_c g_f\frac{M+1}{M}F_2(\omega)\left[\frac{\epsilon}{d}+\frac{n_d+n_o/A_o}{(M+1)/M}\right] \qquad (4\text{-}6)$$

Using the same kind of filter as used in Chapter 3,

$$F_2(\omega)=\frac{\omega_o^2+\sqrt{2}\,\omega_o j\omega}{\omega_o^2+g_o j\omega}\doteq\frac{\omega_o^2+\sqrt{2}\,\omega_o j\omega}{g_o j\omega}$$

$$g_o=kA_o g_c g_f$$

$$\frac{\hat{T}}{d}=\frac{(\omega_o^2+\sqrt{2}\,\omega_o j\omega)}{-\omega^2}\left[\frac{\epsilon}{d}+\frac{n_d+n_o/A_o}{(M+1)/M}\right] \qquad (4\text{-}7)$$

The closed loop function $H(\omega)$ is

$$H(\omega)=\frac{\omega_o^2+\sqrt{2}\,\omega_o j\omega}{-\omega^2+\sqrt{2}\,\omega_o j\omega+\omega_o^2} \qquad (4\text{-}8)$$

$$\frac{\hat{T}}{d}=H(\omega)\left[\frac{T}{d}+\frac{n_d+n_o/A_o}{(M+1)/M}\right] \qquad (4\text{-}9)$$

The value of $\hat{T}/d$ without noise is

$$\frac{\hat{T}}{d}=H(\omega)\frac{T}{d}$$

$$\frac{\epsilon}{d}=\frac{T-\hat{T}}{d}=\frac{T}{d}(1-H(\omega))=\frac{T}{d}\left\{\frac{-\omega^2}{\omega_o^2+\sqrt{2}\,\omega_o j\omega-\omega_o^2}\right\} \qquad (4\text{-}10)$$

Letting $\frac{T(t)}{d}=at$

$$\frac{T(\omega)}{d}=\frac{a}{-\omega^2}$$

$$\frac{\hat{T}(t)}{d}=at-\frac{\sqrt{2}\,a}{\omega_o}e^{-\frac{\omega_o}{\sqrt{2}}t}\sin\frac{\omega_o}{\sqrt{2}}t \qquad (4\text{-}11)$$

$$\frac{\epsilon(t)}{d}=\frac{\sqrt{2}\,a}{\omega_o}e^{-\frac{\omega_o}{\sqrt{2}}t}\sin\frac{\omega_o}{\sqrt{2}}t \qquad (4\text{-}12)$$

$$\max\frac{\epsilon(t)}{d}=\frac{a}{2.19\,\omega_o} \qquad \text{at } t=\frac{\pi}{2\sqrt{2}\,\omega_o} \qquad (4\text{-}13)$$

The critical value of ω_o for expected maximum value of a is to be decided from nonlinear analysis. If we limit this as

max. $\epsilon/d\leq 1$

then $a/\omega_o\leq 2.19$ (4-14)

$$E_T^2 = \int_0^\infty \epsilon(t)^2 \, dt = \frac{d^2 a^2}{2\sqrt{2}\,\omega_0^3} \qquad (4\text{-}15)$$

B) Acquisition transient

If we take $D_d(\epsilon)$ only, from the previous equation,

$$\frac{\hat{\tau}}{d} = \frac{g_0(\omega_0^2 + \sqrt{2}\,\omega_0 j\omega)}{j\omega\,\omega_0^2 + g_0(j\omega)^2} D_d(\epsilon)\frac{M}{M+1}$$

$$= \frac{1 + \sqrt{2}\,j\omega/\omega_0}{\frac{\omega_0}{g_0}\frac{j\omega}{\omega_0} + \left(\frac{j\omega}{\omega_0}\right)^2} D_d(\epsilon)\frac{M}{M+1} \qquad (4\text{-}16)$$

Let $x = \epsilon/d \qquad y = T/d \qquad y-x = \hat{\tau}/d \qquad \tau = \omega_0 t \qquad g_0/\omega_0 = g$

$\frac{j\omega}{\omega_0}$ implies $\frac{1}{\omega_0}\frac{d}{dt} = \frac{d}{d\tau}$

$D_d(\epsilon)\frac{M}{M+1}$ is denoted $D(x)$

$$D(x) = \begin{cases} 2-x & 1 \leqslant x \leqslant 2 \\ x & -1 \leqslant x \leqslant 1 \\ -2-x & -2 \leqslant x \leqslant -1 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{dD}{dx} = D'(x) = \begin{cases} 1 & -1 \leqslant x \leqslant 1 \\ -1 & -2 \leqslant x \leqslant -1 \quad \text{or} \quad 1 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

then $$\left(\frac{1}{g}\frac{d}{d\tau} + \frac{d^2}{d\tau^2}\right)(y-x) = \left(1 + \sqrt{2}\frac{d}{d\tau}\right) D(x) \qquad (4\text{-}17)$$

$$\ddot{y} + \dot{y}/g = \ddot{x} + \dot{x}/g + D(x) + \sqrt{2}D'(x)\dot{x}$$

where $\dot{x} = \frac{dx}{d\tau}$ $\qquad \ddot{x} = \frac{d^2x}{d\tau^2}$ etc.

$$\ddot{x} + \left(\frac{1}{g} + \sqrt{2}D'(x)\right)\dot{x} + D(x) = \ddot{y} + \dot{y}/g \qquad (4\text{-}18)$$

The slope of phase trajectory γ is

$$\gamma = \frac{\ddot{x}}{\dot{x}} = \frac{d\dot{x}}{dx} = -\frac{D(x) + (\sqrt{2}D'(x) + 1/g)\dot{x} - \dot{y}/g - \ddot{y}}{\dot{x}} \qquad (4\text{-}19)$$

If we plot this performance on the phase plane $\dot{x}$ and x, $D'(x)$ and $D(x)$ are the function of only x and we can find the dividing line on $x = -2$, $x = -1$, $x = 1$ and $x = 2$. Let these regions be called A, B, C, D, E as shown in Fig. 4-3.

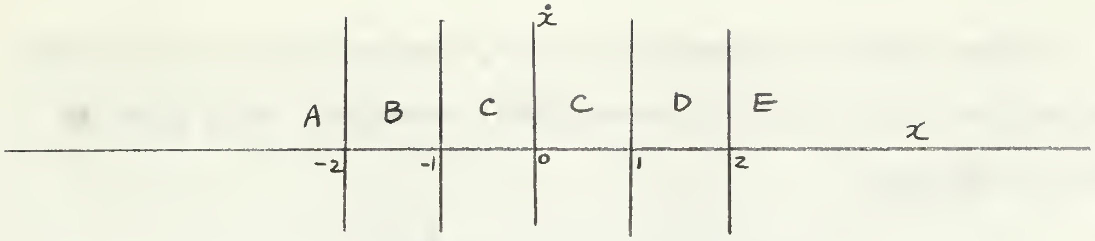


Fig. 4-3 Regions in Phase Trajectory

Region A and E

$$\left.\begin{aligned} & D'(x) = D(x) = 0 \\ & \ddot{x} + \frac{1}{g}\dot{x} = \ddot{y} + \frac{1}{g}\dot{y}, \qquad \gamma = -\frac{\frac{1}{g}\dot{x} - \dot{y}/g - \ddot{y}}{\dot{x}} \\ & \dot{x} = \frac{\dot{y}/g + \ddot{y}}{\gamma + 1/g} \end{aligned}\right\} \qquad (4\text{-}20)$$

γ does not contain x, but is defined only by $\dot{x}$ and the input.

Thus, if $\dot{x} = \dot{y}$ at the initial point, then

$$\gamma = \ddot{y}/\dot{x}$$

$$\dot{x} = \frac{\ddot{y} + \frac{1}{g}\dot{y}}{\ddot{y} + \frac{1}{g}\dot{x}}\dot{x}$$

$$\dot{y} = \dot{x}$$

$$x = y + x_0 \qquad x_0: \text{ initial position} \qquad (4\text{-}21)$$

If $x \neq y$ at $\tau = 0$, letting the initial values of x, $\dot{x}$, y, $\dot{y}$ be x_0, $\dot{x}_0$, 0, $\dot{y}_0$, and substitute them into the general solution of this equation

$$x = c_1 + c_2\, e^{-\tau/g} + y$$

and we obtain

$$x = y + x_0 + g(\dot{x}_0 - \dot{y}_0)(1 - e^{-\tau/g}) \qquad (4\text{-}22)$$

Region B

$$\left.\begin{aligned} & D'(x) = -1 \\ & D(x) = -2 - x \\ & \gamma = -\frac{-2 - x + (1/g - \sqrt{2})\dot{x} - \dot{y}/g - \ddot{y}}{\dot{x}} \\ & \dot{x} = \frac{(2+x) + \dot{y}/g + \ddot{y}}{\gamma + 1/g - \sqrt{2}} \end{aligned}\right\} \qquad (4\text{-}23)$$

For a constant velocity input, i.e. $\ddot{y} = 0$, the point having a constant $\gamma = k$ draws a line.

$$(k + \frac{1}{g} - 2)\dot{x} - (x + 2 + \dot{y}/g) = 0$$

If g is large enough to approximate $1/g = 0$, then upper part of the line combining ($x = -1$, $\dot{x} = -0.518$), ($x = -2$, $\dot{x} = 0$) converges. This line is derived as follows,

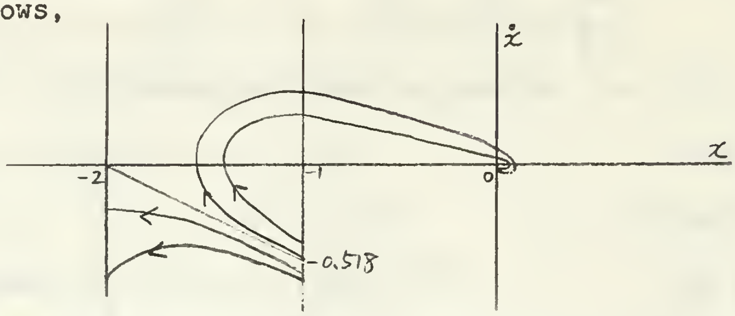


Fig. 4-4 Phase Trajectory in Region B

for constant $\gamma = k$

$$(k-\sqrt{2})\dot{x} - (x+2) = 0$$

$$\dot{x}\Big|_{x=-1} = -\frac{2+x}{\sqrt{2}-k}\Big|_{x=-1} = -\frac{1}{\sqrt{2}-k} \qquad k<0$$

the slope of the line $= k = \dfrac{\dot{x}_0}{1} = -\dfrac{1}{\sqrt{2}-k}$

therefore $\quad k^2 - \sqrt{2}\,k - 1 = 0$

$$k = -\frac{\sqrt{3}-1}{\sqrt{2}}$$

$$\dot{x}_0 = -\frac{1}{\sqrt{2}+\frac{\sqrt{3}-1}{\sqrt{2}}} = -0.518$$

Region C

$$\left.\begin{aligned} D(x) &= x \\ D'(x) &= 1 \\ \gamma &= -\frac{x + [\sqrt{2} + 1/g]\dot{x} - \dot{y}/g - \ddot{y}}{\dot{x}} \\ \dot{x} &= \frac{-x + \dot{y}/g + \ddot{y}}{\gamma + 1/g + \sqrt{2}} \end{aligned}\right\} \qquad (4\text{-}24)$$

Region D

$$\left.\begin{aligned} D(x) &= 2-x \\ D'(x) &= -1 \\ \gamma &= -\frac{2 - x + (-\sqrt{2} + 1/g)\dot{x} - \dot{y}/g - \ddot{y}}{\dot{x}} \\ \dot{x} &= \frac{-2 + x + \dot{y}/g + \ddot{y}}{\gamma + 1/g - \sqrt{2}} \end{aligned}\right\} \qquad (4\text{-}25)$$

The line which divides the plane into converging and diverging parts is derived by the same way as that in region B.

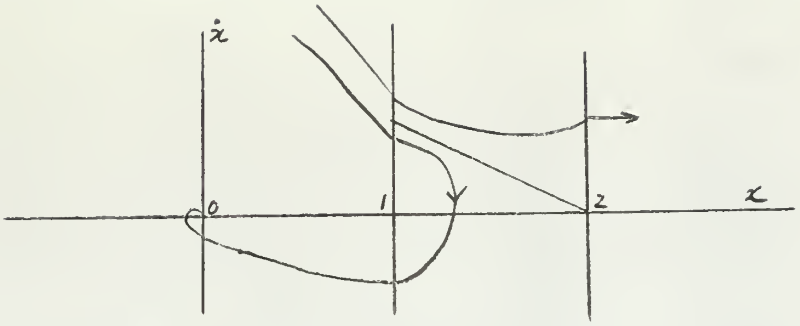


Fig. 4-5 Phase Trajectory in Region D

The result obtained by CDC 1604 digital computer for $g = \infty$ and several initial conditions is shown in Fig. 4-6, (Computer programming, see Appendix II).

From this figure $\dot{x}=2.35$, $x=0$ at the initial condition is the limit to converge. It implies that $\frac{a}{\omega_0} \leqslant 2.35$.

Time response of position error and velocity error are shown in Fig. 4-7, 8. From these figures the locking time is approximately $6/\omega_0 \doteq 2\pi/\omega_0$

The phase trajectory, x vs. τ and $\dot{x}$ vs. τ for g=10 are shown in Fig. 4-9, 10, 11. From these figures we can find the position error of 0.2, and the limit of $\dot{y}$ is approximately 2.0. Lock time is also $6/\omega_0 \doteq 2\pi/\omega_0$.

4.2 Noise and its Effect on the tracking

In this section, I would first like to discuss the effect of self-noise; second the effect of the input Gaussian noise, and finally, the effect of the limiter before this stage.

a) Self-noise

Self-noise is a function of time and phase error. Continuing from the last equation (4-2), power spectral density for the time error ϵ being an integer multiple of d is derived as follows;

$\underline{n_\Delta(t,kd), \text{ where } k=0}$

The autocorrelation function of this self-noise is the sum of three triangular waves as shown. By the Wiener-Khinchin relation the power

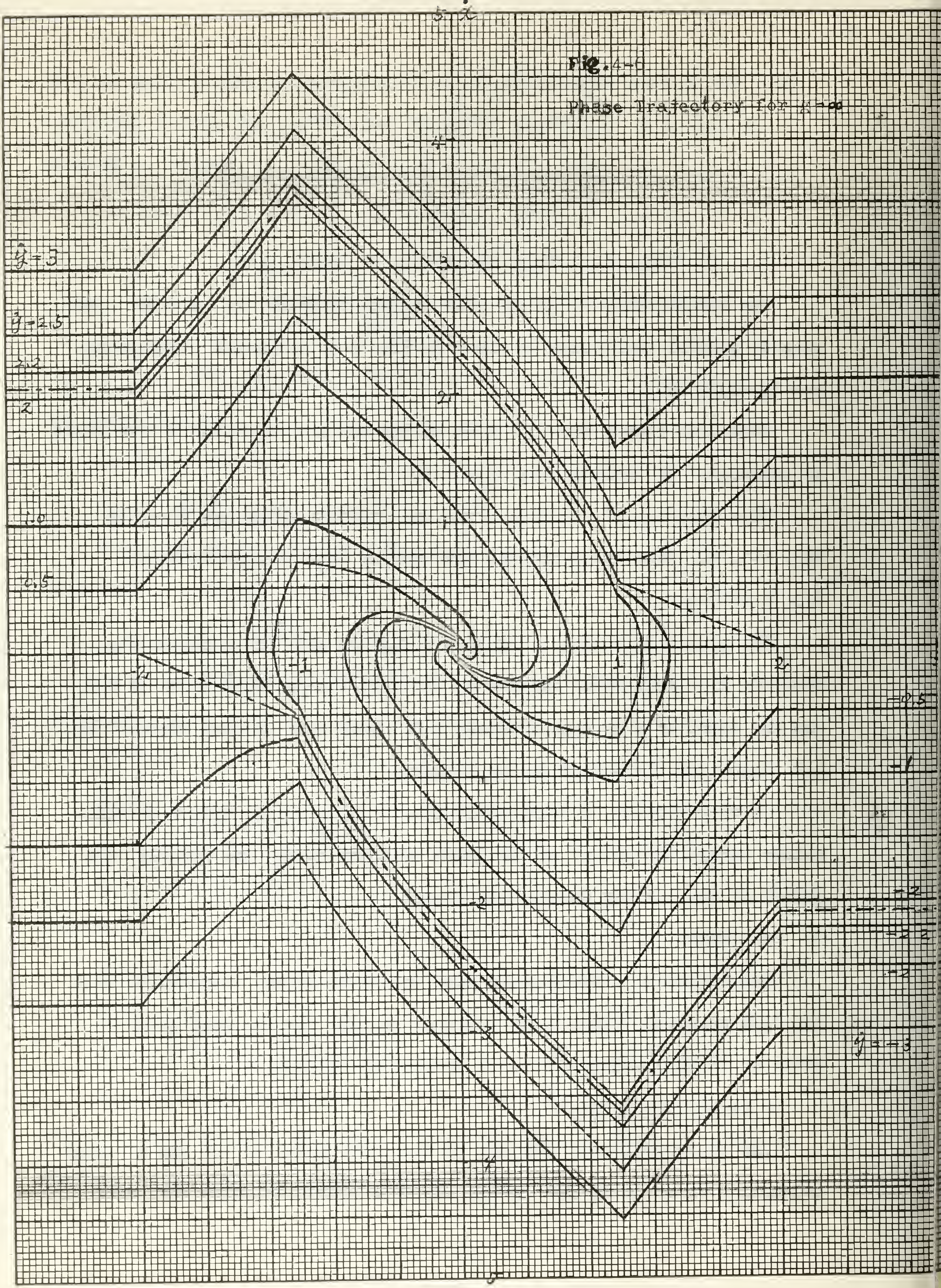


Fig. 4-6

Phase Trajectory for K = ∞

K✠E 10 X 10 TO ½ INCH 7½ X 10 INCHES KEUFFEL & ESSER CO. 46 1475 MADE IN U.S.A.

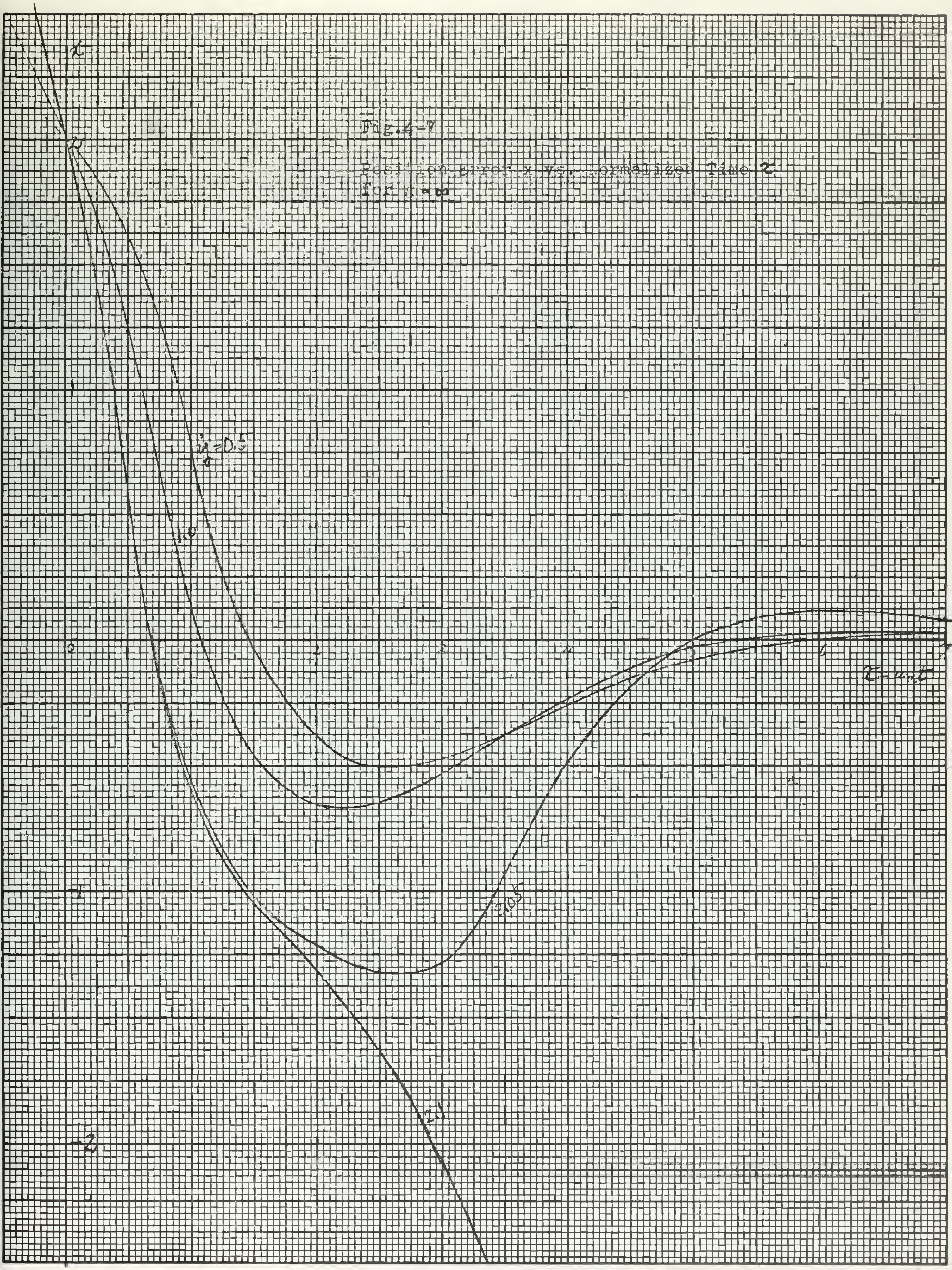


Fig. 4-7

Position Error x vs. Normalized Time τ for x = ∞

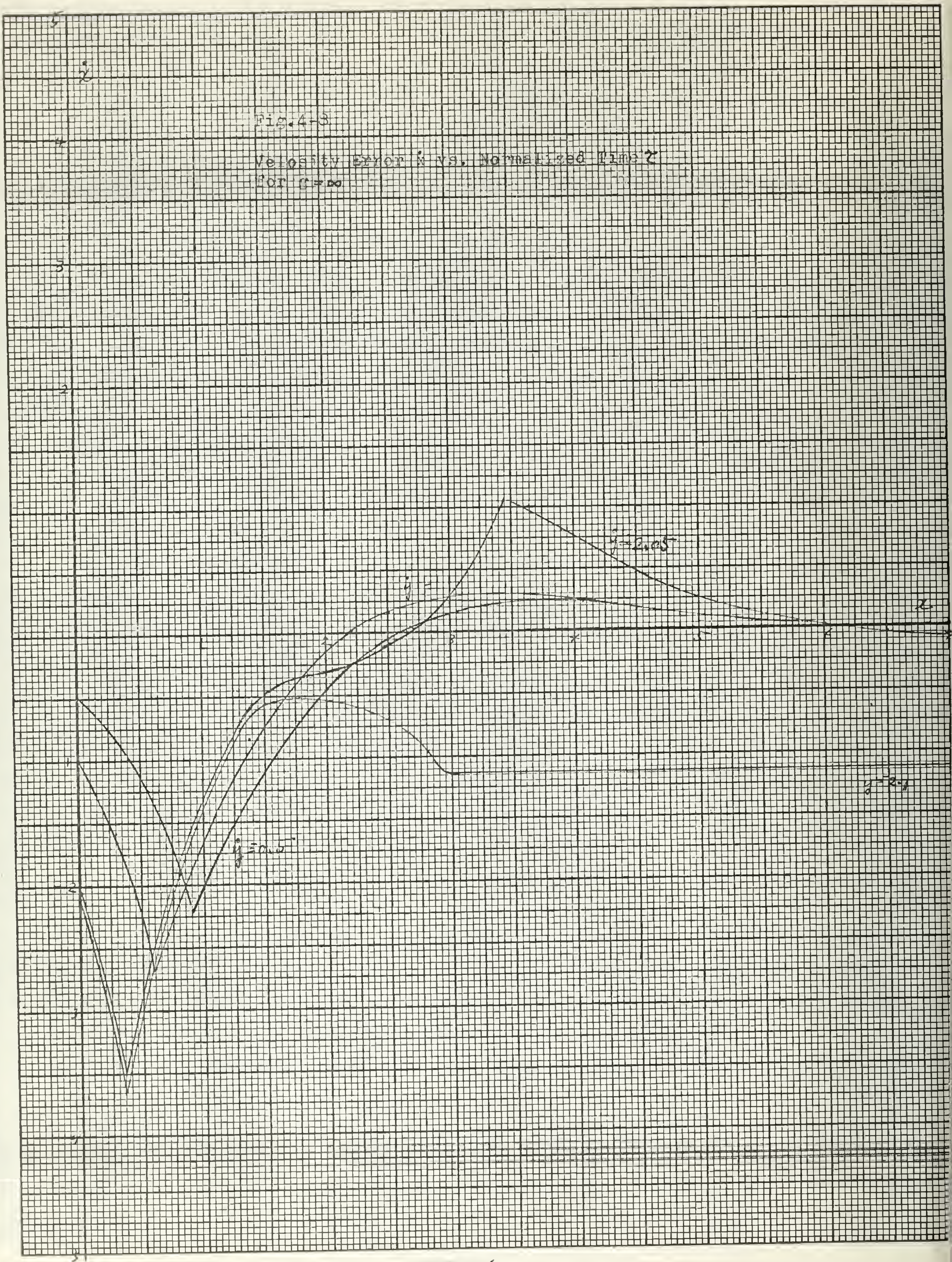


Fig.4-8

Velocity Error $\dot{x}$ vs. Normalized Time τ For $g=\infty$

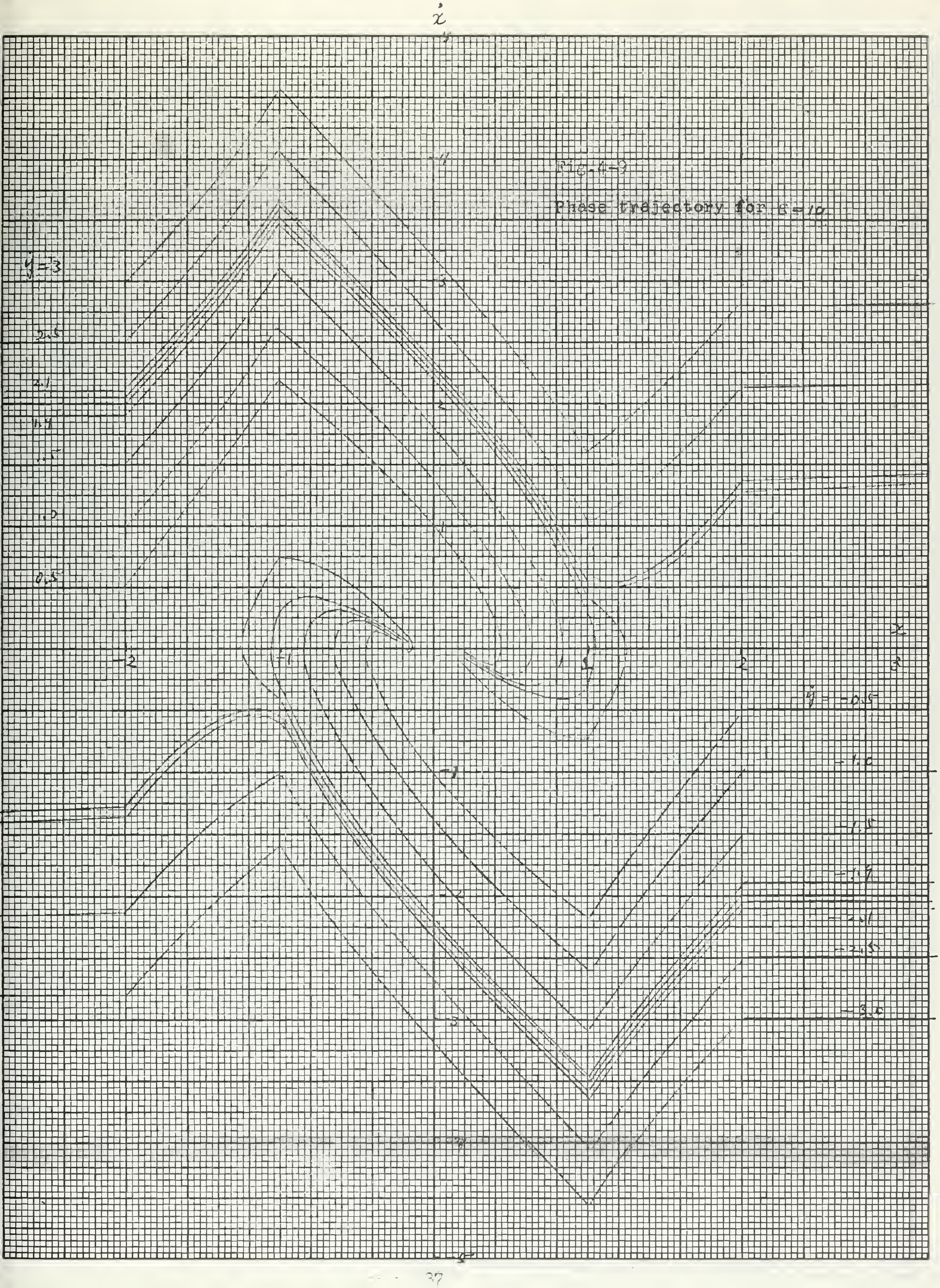


Fig.4-9

Phase trajectory for g=10

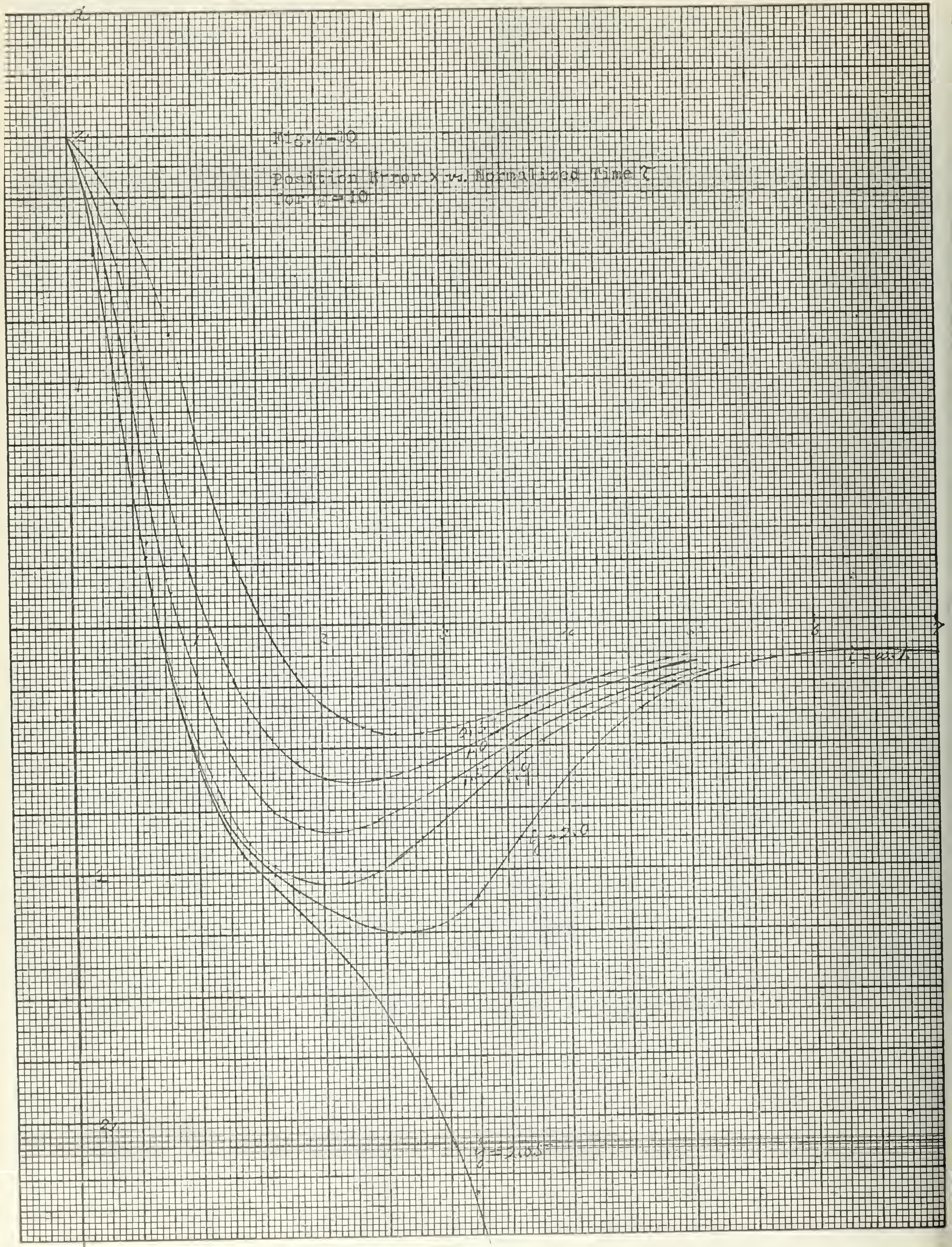


Fig. 4-10

Position Error x vs. Normalized Time τ for $p = 10$

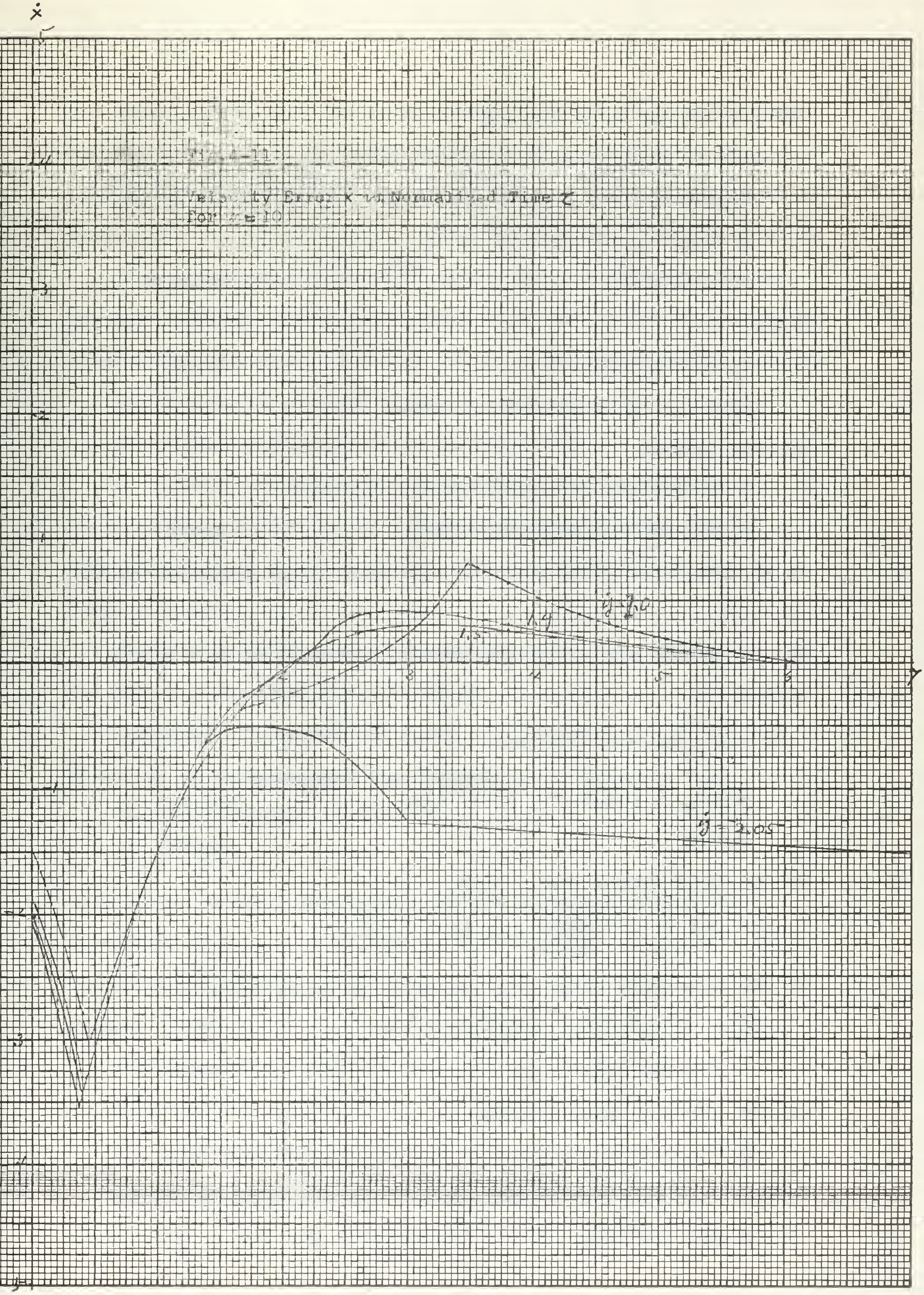
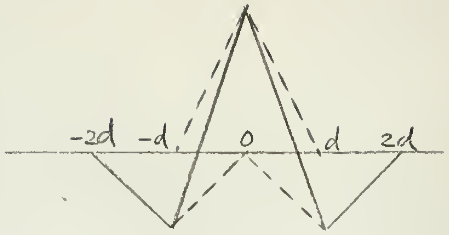


Fig. 4-11

Velocity Error $\dot{x}$ vs Normalized Time τ
For [illegible] = 10

spectral density is the Fourier transform of this wave.



$$G_{n_s}(\omega, \epsilon=0)$$
$$= \frac{1}{M}\left(\frac{\sin \omega d/2}{\omega d/2}\right)^2 \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \delta\left(\omega-\frac{2\pi n}{Md}\right)\left\{2-e^{-j\omega d}-e^{j\omega d}\right\}$$
$$= 2G_s(\omega)(1-\cos\omega d) \qquad (4-26)$$

k=±1

$$G_{n_s}(\omega, \epsilon=\pm d) = G_s(\omega) \qquad (4-27)$$

Since $n_s(t, d)$ is the same as signal.

k=other integer

By transforming the autocorrelation function

$$G_{n_s}(\omega, \epsilon=kd) = 2G_s(\omega)\left[1-\cos(r-q)\omega d\right] \qquad (4-28)$$

We are interested in the effect of this noise in the linearized region $-d \leqslant \epsilon \leqslant d$.

For the extreme case $\epsilon=\pm d$, the power of self-noise in the output of $H(\omega)$ is

$$\sigma_{n_s}^2 = \frac{d^2}{2\pi}\int_{-\infty}^{\infty} G_{n_s}(\omega, d)\,|H(\omega)|^2\, d\omega$$
$$= \frac{d^2}{2\pi}\int_{-\infty}^{\infty} \frac{1+2(\omega/\omega_0)^2}{1+(\omega/\omega_0)^4}\,\frac{1}{M}\left(\frac{\sin \omega d/2}{\omega d/2}\right)^2 \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \delta\left(\omega-\frac{2\pi n}{Md}\right) d\omega$$
$$\doteq d^2\omega_0 d \qquad (4-29)$$

For the least extreme case, $\epsilon=0$

$$\sigma_{n_s}^2 = \frac{4d^2}{M}\sum_{n=-\infty}^{\infty} \frac{1+2\left(\frac{2\pi n}{Md\omega_0}\right)^2}{1+\left(\frac{2\pi n}{Md\omega_0}\right)^4}\left(\sin\frac{\pi n}{M}\right)^2\left(\frac{\sin\frac{\pi n}{M}}{\pi n/M}\right)^2$$

b) The mean-square delay error caused by the input noise

$$\sigma_{n_g}^2 = d^2\left(\frac{M}{M+1}\right)^2 \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{G_{n_g}(\omega)}{A_0^2}\,|H(\omega)|^2\, d\omega$$
$$= 3.08 d^2\left(\frac{M}{M+1}\right)^2 \frac{\eta\,\omega_0}{A_0^2} \qquad (4-30)$$

4.3 The effect of input limiting

By the previous discussion we approximated the input noise as Gaussian with zero mean and the variance σ_n^2. The signal is a square wave of amplitude $\pm A_0$.

Given a digit which has equal probability of +A or -A with additive Gaussian noise, and if we decide the signal is ±A according to the polarity of input, the error probability for each digit is derived as follows:

When we defined $A=\sqrt{\frac{1}{2}(1+e^{-\sigma_{\theta_0}/2})}\ A\sin\theta$, this was derived as an averaged value for some interval, hence if we want to get the probability of error for each digit, we can not use A_0 as the amplitude of the signal.

The probability of error P_E is

$$P_E = P(s(t)=+1)P(\text{output is negative}/s(t)=+1)$$

$$P(s(t)=-1)P(\text{output is positive}/s(t)=-1) \qquad (4\text{-}31)$$

The conditional probability of each term is the same value, because additive noise has zero mean and the signal amplitude is the same in absolute value.

$$P_E = 2P(s(t)=+1)P(\text{output is negative}/s(t)=+1) \qquad (4\text{-}32)$$

The output of the first discriminator is

$$A\cos\theta\sin\theta_0 + As(t)\sin\theta\cos\theta_0 + n_g(t)$$

θ_0 and n_g are Gaussian, with zero mean and

$$\sigma_{\theta_0}^2 = \frac{3\eta B_0}{4\sqrt{2}\ A^2\cos^2\theta} + \theta^2 B_0 d$$

$$\sigma_{n_g}^2 = \eta\frac{2\pi}{d}$$

they are not mutually independent, but the normalized correlation co-efficient ρ is

$$\rho = \frac{B_0\eta}{A\cos\theta}\Big/\sigma_{\theta_0}\sigma_{n_g}$$

From eq. (4-32)

$$P(\text{output is negative}/s(t)=+1)$$

$$= P\left[A\sin(\theta+\theta_0)+n_g<0\right]$$

$$= P\left[n_g \leqslant -A\sin(\theta+\theta_0)\right]$$

$$= \frac{1}{2\pi\sigma_\theta\sigma_n\sqrt{1-\rho^2}}\int_{\theta_0=-\infty}^{\infty}\int_{n_g=-\infty}^{-A\sin(\theta+\theta_0)} exp\left\{-\frac{1}{2(1-\rho^2)}\left(\frac{\theta_0^2}{\sigma_{\theta_0}^2}-\frac{2\rho\theta_0 n_g}{\sigma_{\theta_0}\sigma_n}+\frac{n_g^2}{\sigma_{n_g}^2}\right)\right\}dn_g\,d\theta_0$$

$$\doteq \frac{1}{\sqrt{2\pi}\ \sigma_{\theta_0}}\int_{-\infty}^{\infty} exp\left(-\theta_0^2/\sigma_{\theta_0}^2\right) erf\left\{\frac{1}{\sqrt{1-\rho^2}}\left(-\frac{A}{\sigma_n}\sin(\theta+\theta_0)-\frac{\rho\theta_0}{\sigma_{\theta_0}}\right)\right\}d\theta_0 \quad (4\text{-}33)$$

where $\operatorname{erf}(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-x^2/2}dx$

Consider a pulse train of m digits, where each digit has probability of error P_E, then if all pulses are 1, the average of this pulse train is derived as follows.

The number of error k in m digits distributes binormally, and if m is large enough and mP_E is not small, then this distribution can be approximated by a normal distribution with mean $= mP_E$, variance $mP_E(1-P_E)$. In this case each error causes the decrease of 2/m in the average, and this decrease is normally distributed.

The decrease of the averaged voltage v is, thus

$$p(v)=\frac{1}{\sqrt{2\pi}\sqrt{\frac{4P_E(1-P_E)}{m}}}\ e^{-(v-2P_E)^2/2\left(\frac{4P_E(1-P_E)}{m}\right)} \qquad (4\text{-}34)$$

If the pulse train is composed with the same number of +1 and -1, i.e. m/2 of positive pulses and m/2 of negative pulses, then an error which occurred in positive pulses causes negative error 2/m in the average voltage and the same for an error in negative pulses.

x: positive error voltage caused by errors in negative pulses

y: negative error voltage caused by errors in positive pulses

Since the error for each digit is independent, x and y are independent because they are the error caused by only negative or positive pulses.

$$p(x)=\frac{1}{\sqrt{2\pi}\sqrt{2P_E(1-P_E)/m}}\exp\left\{-\frac{(x-P_E)^2}{2(2P_E(1-P_E)/m)}\right\}$$

$$p(y)=\frac{1}{\sqrt{2\pi}\sqrt{2P_E(1-P_E)/m}}\exp\left\{-\frac{(y+P_E)^2}{2(2P_E(1-P_E)/m)}\right\}$$

Since they are independent, the probability distribution of $u=(x+y)$ is

$$\begin{aligned}
p(u)&=\int_{-\infty}^{\infty}p_x(x)\,p_y(u-x)\,dx\\
&=\frac{1}{2\pi(2P_E(1-P_E)/m)}\int_{\infty}^{\infty}\exp\left\{-\frac{1}{2\frac{P_E(1-P_E)}{m}}\left((x-P_E)^2+(u+P_E-x)^2\right)\right\}dx\\
&=\frac{1}{2\pi\sigma^2}\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2\sigma^2}(z^2+(u-z)^2)\right\}dz \quad \text{where} \quad x-P_E=z \quad \frac{2P_E(1-P_E)}{m}=\sigma^2\\
&=\frac{1}{\sqrt{2\pi}\sqrt{2\sigma^2}}\exp\left\{-\frac{u^2}{2(2\sigma^2)}\right\}\\
&=\frac{1}{\sqrt{2\pi}\sqrt{\frac{4P_E(1-P_E)}{m}}}\exp\left\{-\frac{u^2}{2\left(\frac{4P_E(1-P_E)}{m}\right)}\right\} \qquad (4\text{-}35)
\end{aligned}$$

This variance is equal to that in the positive pulse train.

The effects these digit errors cause in the next binary discriminator, will be discussed in two phases, first at the locked point, and this can be considered as a phase jitter in the phase-locked position, and second at the limit where the phase error becomes ±d. This may be considered as the modification of $D_d(\epsilon)$ curve. These two effects will show the analysis of the system.

When one digit error occurs in the input, how does this appear in the outputs of two crosscorrelators? If the system is locked in phase, $\epsilon = 0$, the outputs are also a cyclic permutation of that m-sequence and their phases differ by 1 digit. Since the autocorrelation function of this sequence is nearly zero for $\epsilon = \pm d$, the errors introduced in the outputs of the crosscorrelators do not have any relation to their polarity except the fact that they occurred at the same time. About the polarity, from the characteristics of the randomness of m-sequence, the probability that both errors are positive or negative is $\frac{1}{4}$ each, and the probability the sum is zero is $\frac{1}{2}$. This implies the error probability is decreased by a factor of 2, and we can use $P_E/2$ instead of P_E, and the change in the average voltage output caused by one error becomes double.

Thus, we can modify the equation of the noise voltage as follows.

$$p(u) = \frac{1}{\sqrt{2\pi}\sqrt{\frac{16(P_E/2)(1-P_E/2)}{m}}} \exp\left\{-\frac{u^2}{2\left(\frac{16(P_E/2)(1-P_E/2)}{m}\right)}\right\} \qquad (4\text{-}36)$$

For the case of $\epsilon = d$, one error always appears as negative, and another as positive of negative. This time we do not need to modify P_E, but we can add two effects.

$$p(u) = \frac{1}{\sqrt{2\pi}\sqrt{\frac{8P_E(1-P_E)}{m}}} \exp\left\{-\frac{(u-2P_E)^2}{2\left(\frac{8P_E(1-P_E)}{m}\right)}\right\}$$

We have been discussing the effect of noise on the average through m digits so far, and now we have to make sure of the meaning of the average.

The inverse Fourier transform of $H(\omega)$, $h(t)$ is

$$h(t)=\sqrt{2}\,\omega_0 e^{-\frac{\omega_0}{\sqrt{2}}t}\cos\frac{\omega_0}{\sqrt{2}}t$$

The response of this filter to a pulse of height 1 and duration d is, if we take the response after time d,

$$\frac{\omega_0 d}{\sqrt{2}}e^{-\frac{\omega_0 t}{\sqrt{2}}}\sqrt{2}\sin\left(\frac{\omega_0}{\sqrt{2}}t+\frac{\pi}{4}\right)$$

Since the chance of error is very small, it will be improbable that the digit in error will be discovered. Therefore we can neglect the response of this filter in the digit duration and assume this as an impulse response.

When we look backward for a time interval of T, standing at an arbitrary time t_1, if we use a real averaging device, the value at t_1 is, assuming there are k impulses to occur in T,

$$u_1=\frac{1}{T}\int_{t_1-T}^{t_1}\sum_{i=1}^{k}d\;\delta(t-t_i)\,dt=\frac{kd}{T}\,.$$

If we use $H(\omega)$ as an averager, the output average differs depending upon the positions of impulses, but when we think of the interval of each impulse, this is approximately distributed exponentially, and after taking many intervals T containing k impulses, we find k impulses are distributed in equal intervals as an average.

Thus, the output average is the integration of that response. The time interval T should be determined for $H(\omega)$, and looking at the response $\frac{\omega_0 d}{\sqrt{2}}e^{-\frac{\omega_0}{\sqrt{2}}t}\sqrt{2}\sin(\frac{\omega_0}{\sqrt{2}}t+\frac{\pi}{4})$, we can neglect the effect of the tail after $t=\frac{3\pi\sqrt{2}}{4\omega_0}$.

$$T=\frac{3\pi}{4}\frac{\sqrt{2}}{\omega_0} \qquad (4\text{-}37)$$

$$\begin{aligned}u_2&=\frac{kd}{T}\int_0^T\frac{\omega_0}{\sqrt{2}}e^{-\frac{\omega_0}{\sqrt{2}}t}\sqrt{2}\sin\left(\frac{\omega_0}{\sqrt{2}}t+\frac{\pi}{4}\right)dt\\&=\frac{kd\,4\,\omega_0^2}{3\pi\sqrt{2}\sqrt{2}}\int_0^{\frac{3\pi}{4}\frac{\sqrt{2}}{\omega_0}}\left[e^{-\frac{\omega_0}{\sqrt{2}}t}\left(\sin\frac{\omega_0}{\sqrt{2}}t+\cos\frac{\omega_0}{\sqrt{2}}t\right)\right]dt\\&=\frac{4k\omega_0 d}{3\pi\sqrt{2}}=\frac{kd}{\frac{3\sqrt{3}}{4}\frac{\pi}{\omega_0}}=\frac{kd}{T}\end{aligned} \qquad (4\text{-}38)$$

$$m=\frac{3\pi\sqrt{2}}{4}\frac{1}{\omega_0}\Big/d \qquad (4\text{-}39)$$

Thus we can approximate the effect of $H(\omega)$ by the effect of averaging.

CHAPTER V

THRESHOLD FOR THIS SYSTEM

5.1 Summary of the System Performance

When we evaluate the performance of a phase lock system, maximum frequency deviation, or in other words, maximum target velocity that a tracking system can acquire, and transient time for phase lock, probability of cycles slipping and r.m.s. phase error at the steady state will be considered. The problem of the cycles slipping is considered very important for this system, because the acquisition time in A, E region in Fig. 4-3 can be very large. We can not allow cycle slip in the delay lock discriminator. Cycle slip is related to r.m.s. phase error at the steady state. From the experimental results of W. F. Roland, [7] we can find the linearity of $\sigma_{\theta_0}^2$ vs. B_0 up to 20~30 degree, and the expected number of cycle slip per $\frac{2\pi}{B_0}$ is

6.10^{-4} for σ_{θ_0} : 20°

6.10^{-3} for σ_{θ_0} : 25°

$2.35 \cdot 10^{-2}$ for σ_{θ_0} : 30°

If we use equation (3-3), the time for $e^{-\frac{B_0}{\sqrt{2}}t} = 0.1$ is $\frac{2.32\sqrt{2}}{B_0} \approx \frac{\pi}{B_0}$. This is for the initial position error of zero, and we can approximately use twice the value of it, $\frac{2\pi}{B_0}$, as an acquisition time from the initial error of π.

Let the expected number of cycles slip per $2\pi/B_0$ be p_d and the settling time be $2\pi/B_0$, then as an average

$$l_d = p_d \frac{B_0}{2\pi} dM \tag{5-1}$$

is the number of cycles slip in one period. If one cycle of slip occurs, $\frac{2\pi}{B_0}$ in Md is in transient responce and it can cause longer transient period of the next stage.

For the cycles slip in the next stage, Spilker's paper shows that for delay error having $\sigma_n \leq 0.30d$, a probability to lose the locked-on state in the absence of transient errors is negligible from experiments.

While cycles slip determines the limitation for the values of the S/N ratio, θ and the bandwidth parameter B_0, ω_0, the maximum frequency deviation restricts the lower limit of B_0 and ω_0. From the phase trajectory of the transient response of a phase lock loop for a sinusoid by A. Viterbi, the maximum frequency deviation at position error $= 0$ to be acquired is approximately $2.15B_0$ for this phase lock system concerned. If we restrict $\epsilon(t)\Big|_{max} \leq 1$ in the linearized system,

$$\frac{\Delta\omega}{B_0} \leq 2.19 \qquad \text{from eq. (3-6)}$$

or

$$\frac{\Delta\omega}{B_0} \leq 2.15 \tag{5-2}$$

For the delay lock discriminator, from Fig. 4-6 and 4-10,

$$\dot{y} \leq 2$$

This is $\frac{a}{\omega_0} \leq 2.$ (5-3)

Or, using the result of eq. (4-14),

$$\frac{a}{\omega_0} \leq 2.19 \tag{5-4}$$

The transient time for phase lock is decided mainly by B_0 and ω_0, as follows,

$t_{lock} = \frac{2\pi}{B_0}$ for the phase lock loop

$t_{lock} = \frac{2\pi}{\omega_0}$

These transient interval should be limited by the actual situation.

5.2 An Example of the Calculation Procedure

We are going to decide θ, B_0 and ω_0 for a radar using this phase lock system. The specification of this radar is:

Range	500 Miles

Carrier frequency	200 Megacycles
Pulse duration, d	5 Microsecond (2500 feet)
Number of digits in one period	$M \doteq 1200$
	$T = Md = 5$ Milisecond
Maximum target velocity, v	3 Mach, 3000 feet/sec
Input S/N ratio	less than 0 dB

a) Lower limit of B_o,

$$\Delta\omega\Big|_{max} = \frac{2v}{c}\,\omega_c = \frac{6000}{10^9}\cdot 2\pi \cdot 200\times 10^6 \doteq 7550 \doteq 8000 \text{ rad/sec}$$

$$B_o \geqslant \frac{\Delta\omega}{2} \doteq 4000 \text{ rad/sec}$$

for this B_o, $t_{lock} \doteq \frac{2\pi}{B_o} \doteq 1.6$ msec

This will be short enough comparing with a period of $Md = 5$ msec for the first acquisition and we do not need to increase B_o.

For the delay lock discriminator we put

$T(t) = adt$ in eq. (4-11).

$$\frac{dT}{dt} = \frac{2v}{c} = ad$$

$$a = \frac{2v}{c}\,\frac{1}{d} \doteq 1.2$$

$$\frac{a}{\omega_0} \leqslant 2.0$$

$$\omega_o \geqslant 0.6 \text{ rad/sec}$$

but $t_{lock} \doteq \frac{2\pi}{\omega_o}$ (5-5)

for $\omega_o = 0.6$ $t_{lock} = 10$ sec

This is too large for this operation, and we need to make ω_o large in the limit where the cycle slip does not occur.

b) The number of cycle slips per $\frac{2\pi}{B_o}$

$$l_s = p_s \frac{B_o}{2\pi} Md \qquad (5\text{-}6)$$

$B_o = 4000$ $d = 5.10^{-6}$ $M = 1200$

$$l_s = 4p_s$$

Once a cycle slip occurs, it takes 1.6 msec to recover and this becomes a loss of about one third of a period. Thus we want to limit 1_d in 3.10^{-2}, or r.m.s. phase error up to 25°.

c) $\sigma_{\theta_0}^2 = \sigma_{\theta_n}^2 + \sigma_{\theta_d}^2$

$$\sigma_{\theta_n}^2 = \frac{\eta}{2A^2\cos^2\theta}\cdot 1.06B_0 = \frac{1}{2\cos^2\theta}\,\frac{1}{K}\left(\frac{1.06\,B_0}{2\pi\times 400\times 10^3}\right)$$

where K is input S/N ratio

$$\sigma_{\theta_d}^2 = B_0 d\theta^2$$

$\sigma_{\theta_d}^2, \sigma_{\theta_n}^2$ and $\sigma_{\theta_0}^2$ vs. θ for various values of K are shown in Fig. 5-1.

d) $\frac{A_0^2}{A^2} = \frac{1}{2}(1 + e^{-\sigma_{\theta_0}^2/2})\sin^2\theta$

Under the limitation of $\sigma_{\theta_0}^2 \leq 0.19$ $(=(25^\circ)^2)$, we can plot $\frac{A_0^2}{A^2}$ vs. θ for various K, as shown in Fig. 5-2.

From Fig. 5-2, we can find θ which maximizes A_0^2/A^2 with respect to K, and this is in Fig. 5-3. The maximized A_0^2/A^2 vs. K is shown in Fig. 5-4.

e) The mean sequare delay error σ_n^2 is the sum of the contribution of noise and self-noise. Taking the worst value of self-noise, from eq. (4-30) and (4-29),

$$\frac{\sigma_n^2}{d^2} \doteq 3.08\,\frac{\omega_0}{K(A_0^2/A^2)(2\cdot 200\cdot 10^3)} + \omega_0 d \qquad (5\text{-}7)$$

In Fig. 5-5, we showed σ_n^2/d^2 vs. K with maximum value of A_0^2/A^2, for various values of ω_0. In order to avoid cycle slip, we limited $\sigma_n/d \leq 0.2$, and the maximum value of ω_0 vs K to satisfy this limitation, and t_{lock} vs. K for this ω_0 are finally shown in Fig. 5-6.

From these figures, we can see that this system can not show good performance for the S/N ratio less than -15 dB.

5.3 The effect of a limiter before the delay lock discriminator does not improve S/N ratio, because for low S/N ratio, P_E approaches to $\frac{1}{2}$ and phase lock control voltage $D_d(\epsilon)$ effectively decreases to zero.

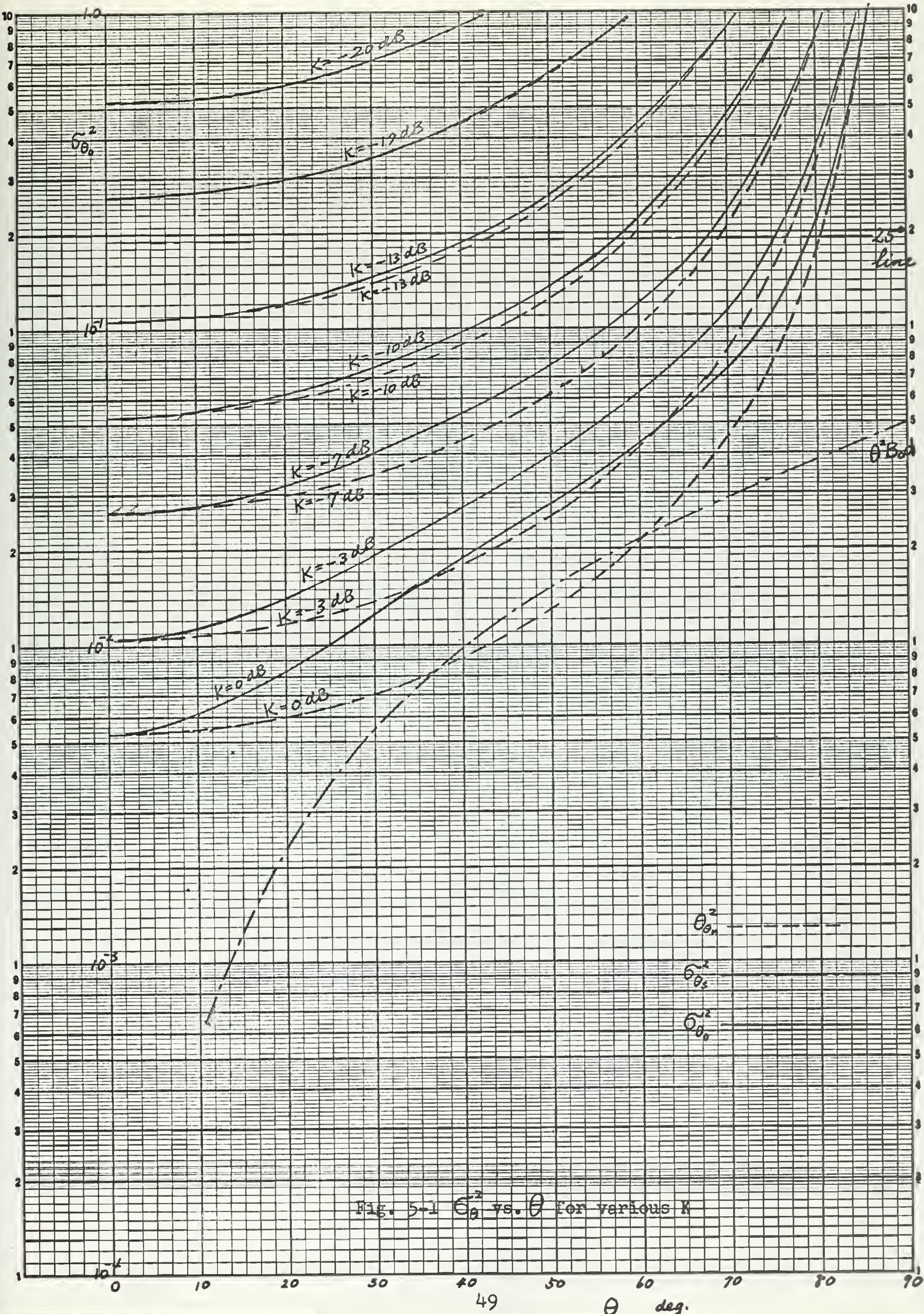


Fig. 5-1 σ_θ^2 vs. θ for various K

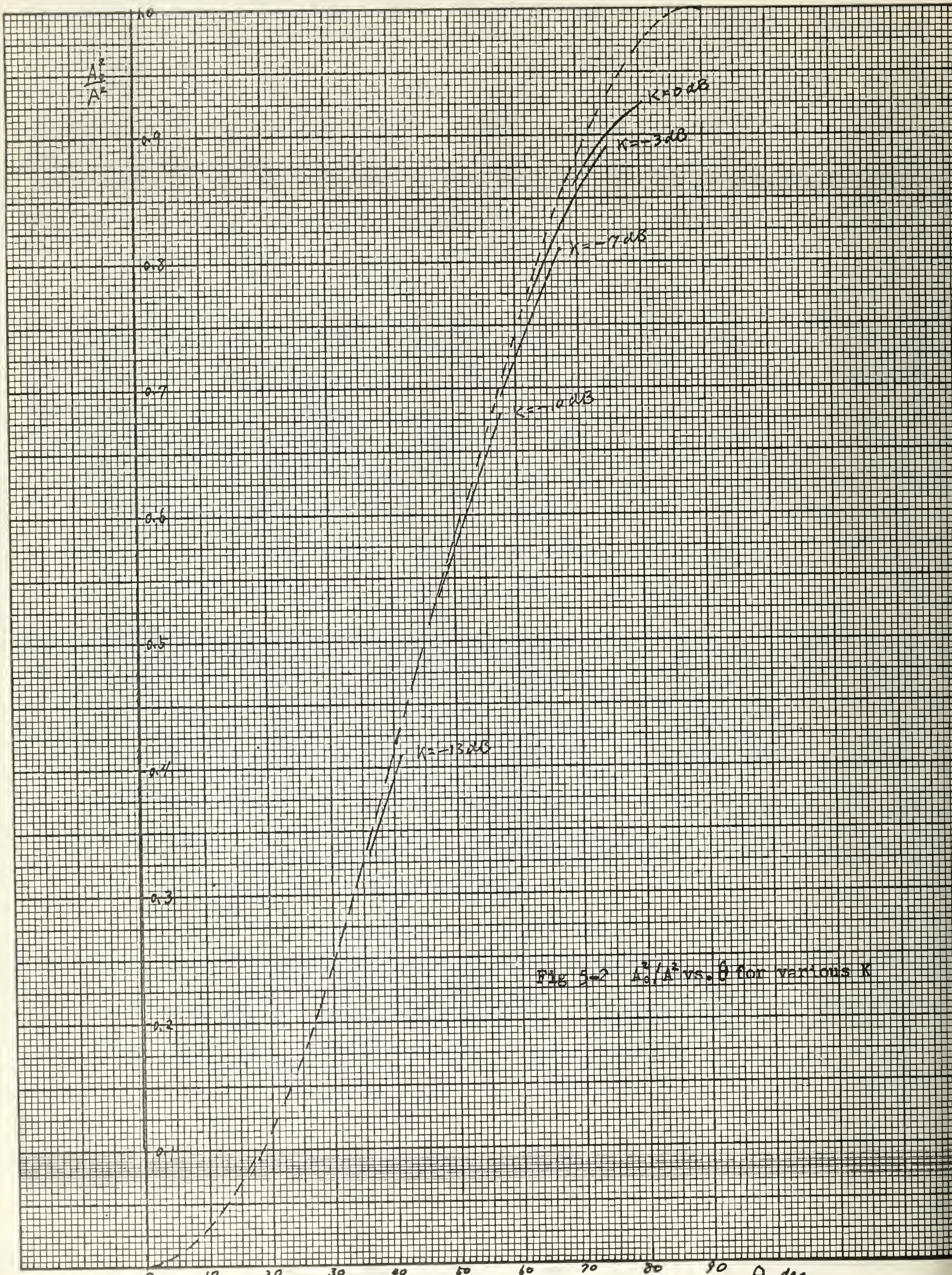


Fig 5-2 A_0^2/A^2 vs. θ for various K

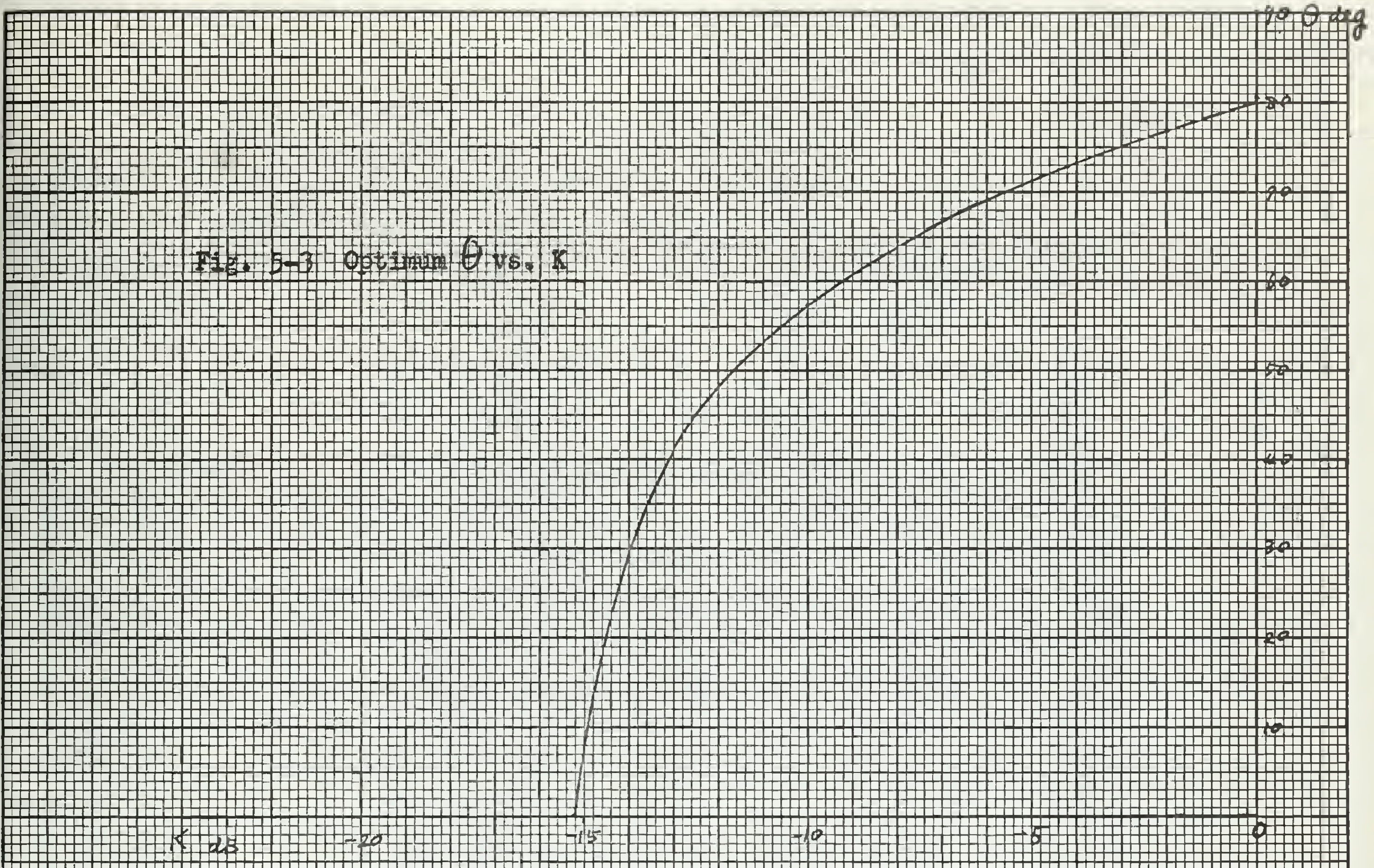


Fig. 5-3 Optimum θ vs. K

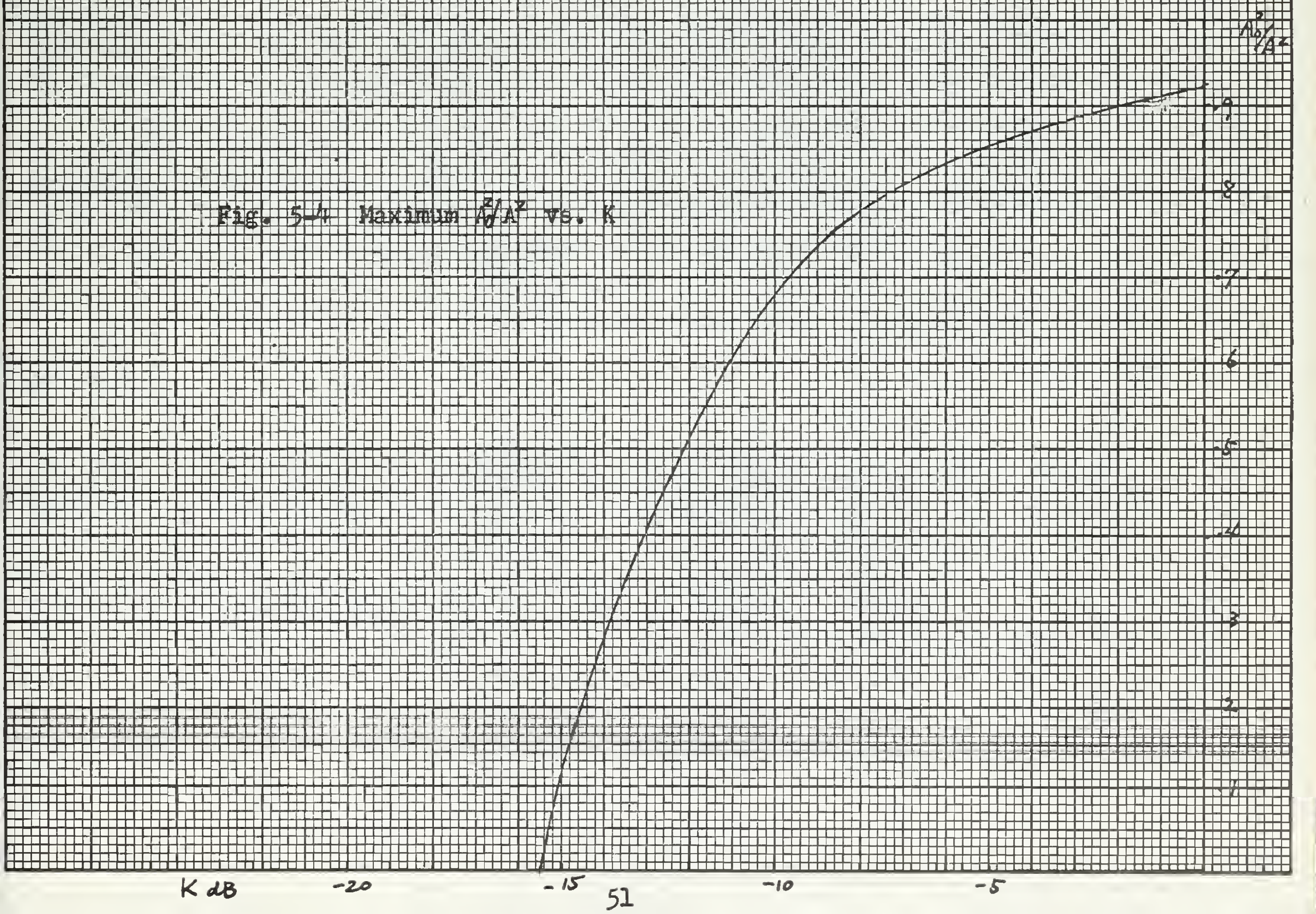


Fig. 5-4 Maximum A_o^2/A^2 vs. K

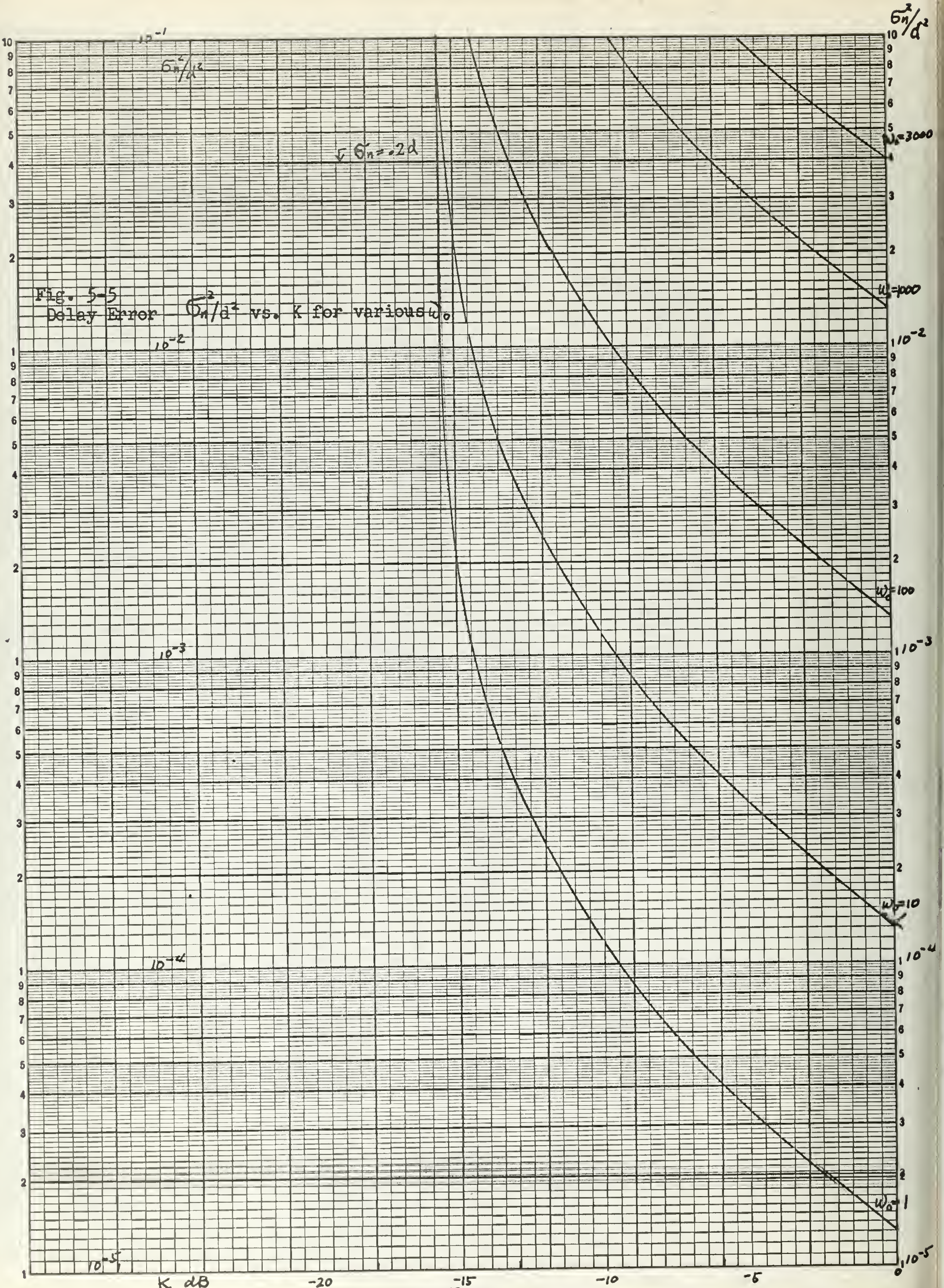


Fig. 5-5
Delay Error σ_n^2/d^2 vs. K for various ω_0

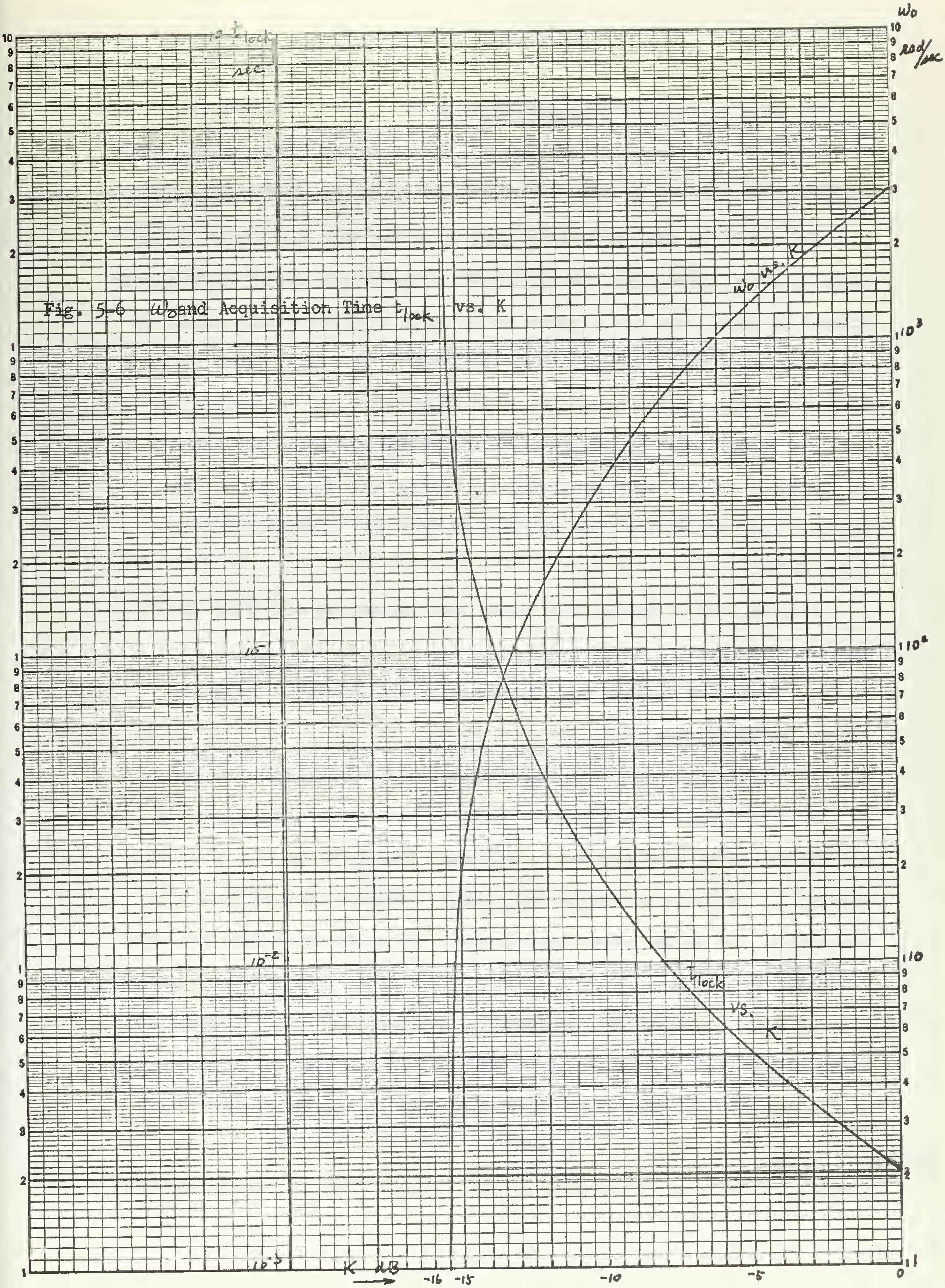


Fig. 5-6 ω_o and Acquisition Time t_{lock} vs. K

CHAPTER VI

CONCLUSION

The result in Chapter 5 shows that if we use the demodulator in the limit to avoid cycle slip, the amplitude of the output signal A_o is mainly decided by $\sin\theta$, and r.m.s. phase error σ_{θ_o} is also determined mainly by the noise input near the upper limit of $25\sim30^\circ$. Eq. (5-7) shows that the variance of delay error σ_n^2 is the sum of input noise and self-noise and both are proportional to ω_o, therefore for better S/N ratio the self-noise is dominant. We are interested more in the low S/N ratio region than in the high one, and we can also approximate σ_n^2 by the contribution of the input noise.

When we want to design this phase lock system, we will find several opposing factors.

1) The phase lock ability over the range of the frequency deviation limits the lower side of B_o, ω_o.

2) The cycle slip bounds the upper limit of θ and B_o, ω_o with respect to K.

3) For a satisfactory acquisition time, we have to get large B_o and ω_o.

4) The effect of d is related to the input bandwidth and ω_o. d is also related to the range resolution of a radar. Small d will offer a good resolution, but on the other hand it costs S/N ratio and limits the input frequency deviation.

We tried to solve the problem for a constant S/N ratio in Chapter 5. Actually input S/N ratio can not be constant in a system application, and we can estimate only the least desirable value or a range of S/N ratio. When we use this system for ranging, σ_n^2 does not have much meaning in itself, but is very important as a cause of cycle slip. Looking through these four factors, 2 and 3 are especially significant in opposing each other, and we will finally decide B_o, and ω_o by applying a weighting factor. The actual designing process must be decided according to each case, and we want to point out the fundamental relations between the above four factors, for they must be further investigated.

CHAPTER VII

ACKNOWLEDGEMENT

I appreciate the assistance of Professor C. F. Klamm, Jr., for his suggestions and advice about PN sequences, and the idea for this system as a whole.

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APPENDIX I

SELF-NOISE

The phase error output by signal s(t) itself is $\theta S(\omega)H(\omega)$ considering the relation shown in Fig. 3-5.

$$\sigma_{\theta_s}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \theta^2 G_s(\omega)|H(\omega)|^2 d\omega$$

$$\doteq \frac{\theta^2}{2\pi}\int_{-\infty}^{\infty} \frac{1+2(\omega/B_0)^2}{1+(\omega/B_0)^4}\frac{1}{M}\left(\frac{\sin \omega d'/2}{\omega d'/2}\right)^2 \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \delta\left(\omega - \frac{2\pi n}{Md'}\right) d\omega$$

$$= \theta^2 \frac{1}{M} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{1+2(2\pi n/Md'B_0)^2}{1+(2\pi n/Md'B_0)^4}\left(\frac{\sin \frac{2\pi n}{B_0 d}\frac{d'}{2}}{\frac{2\pi n}{B_0 d}\frac{d'}{2}}\right)^2$$

$$= \theta^2 \frac{1}{M}\frac{Md'}{2\pi} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{1+2(2\pi n/Md'B_0)^2}{1+(2\pi n/Md'B_0)^4}\left(\frac{\sin \frac{2\pi n}{B_0 d'}\frac{d'}{2}}{\frac{2\pi n}{B_0 d'}\frac{d'}{2}}\right)^2 \frac{2\pi}{Md'}$$

Since M is very large, $\frac{2\pi}{Md}$ is very small and we can approximate this relation by an integral form.

$$\sigma_{\theta_s}^2 \doteq \theta^2 d' \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1+2(\omega/B_0)^2}{1+(\omega/B_0)^4}\left(\frac{\sin \omega d'/2}{\omega d'/2}\right)^2 d\omega \qquad z = \omega/B_0$$

$$= \theta^2 B_0 d' \frac{2}{2\pi}\int_{-\infty}^{\infty} \frac{z^2+1/2}{z^4+1}\left(\frac{\sin B_0 d' z/2}{B_0 d' z/2}\right)^2 dz$$

Let $$I = \frac{2B_0 d'}{2\pi}\int_{-\infty}^{\infty} \frac{z^2+1/2}{z^4+1}\left(\frac{\sin B_0 d' z/2}{B_0 d' z/2}\right)^2 dz$$

$$I = \frac{2B_0 d'}{2\pi}\int_{\infty}^{\infty} \frac{(z+j\frac{1}{\sqrt{2}})(z-j\frac{1}{\sqrt{2}})\frac{1}{2}(1-\frac{1}{2}e^{jB_0 d' z}-\frac{1}{2}e^{-jB_0 d' z})\, dz}{(z-\frac{1}{\sqrt{2}}(1+j))(z-\frac{1}{\sqrt{2}}(-1+j))(z-\frac{1}{\sqrt{2}}(1-j))(z-\frac{1}{\sqrt{2}}(-1-j))(\frac{B_0 d'}{2})^2 z^2}$$

$$I_1 = \frac{2B_0 d'}{2\pi}\frac{1}{2}\int_{-\infty}^{\infty} \frac{z^2+1/2}{(z^4+1)(B_0 d'/2)^2 z^2} dz$$

$$I_2 = \frac{2B_0 d'}{2\pi}\frac{1}{4}\int_{-\infty}^{\infty} \frac{(z^2+1/2)e^{jB_0 d' z}}{(z^4+1)(B_0 d'/2)^2 z^2} dz$$

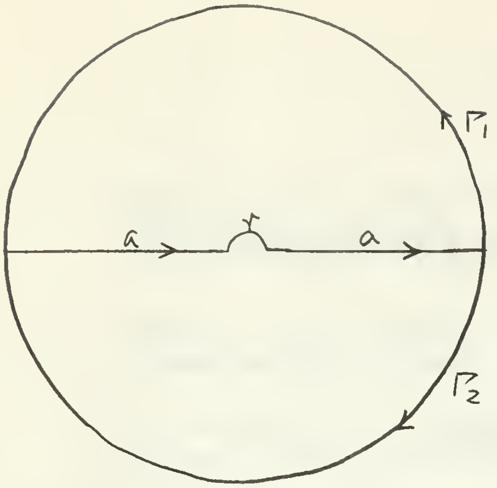
$$I_3 = \frac{2B_0 d'}{2\pi}\frac{1}{4}\int_{-\infty}^{\infty} \frac{(z^2+1/2)e^{-jB_0 d' z}}{(z^4+1)(B_0 d'/2)^2 z^2} dz$$

$$I = I_1 - I_2 - I_3$$

Moreover, let $g(z) = \frac{2B_0 d'}{2\pi}\frac{1}{2}\frac{z^2+1/2}{(z^4+1)(B_0 d'/2)^2 z^2}$

and consider contour integrals along the paths a, r, Γ_1 or Γ_2.

Since $B_0 d$ is positive, the contributions of the integral path Γ_1 to the contour integrals $\int_C g(z)dz$, and $\frac{1}{2}\int_C g(z)e^{jB_0 d' z} dz$, and Γ_2 to $\frac{1}{2}\int_C g(z)e^{-jB_0 d' z} dz$



are all zero. The integral along the path γ does not converge to zero for each integral as $r \to 0$, but the sum

$$\int_\gamma g(z)dz - \int_\gamma \tfrac{1}{2} g(z) e^{jB_o d'z} dz - \int_\gamma \tfrac{1}{2} g(z) e^{-jB_o d'z} dz$$

converges as follows.

$$\lim_{\gamma\to 0}\left\{\int_\gamma g(z)dz - \tfrac{1}{2}\int_\gamma g(z)e^{jB_o d'z}dz - \tfrac{1}{2}\int_c g(z)e^{-jB_o d'z}dz\right\}$$

$$= \lim_{\gamma\to 0}\left\{\frac{1}{2}\frac{2B_o d'}{2\pi}\left[\int_\pi^0 \frac{\frac{1}{2}\gamma j e^{i\theta}}{(B_o d'/2)^2\gamma^2 e^{i2\theta}}d\theta - \frac{1}{2}\int_\pi^0 \frac{\frac{1}{2}\gamma j e^{i\theta}}{(B_o d'/2)^2\gamma^2 e^{i2\theta}}d\theta - \frac{1}{2}\int_\pi^0 \frac{\frac{1}{2}\gamma j e^{i\theta}}{(B_o d'/2)^2\gamma^2 e^{i2\theta}}d\theta\right]\right\}$$

$$= \lim_{\gamma\to 0}\left\{\frac{1}{2}\frac{2B_o d'}{2\pi}\int_\pi^0 \frac{\frac{1}{2}j(1-1)}{(B_o d'/2)^2\gamma e^{i\theta}}d\theta\right\}$$

$$= 0$$

Therefore $I = I_1 - I_2 - I_3$

$$= 2\pi j\left\{\sum_{UHP}(\text{residue for } g(z)) - \tfrac{1}{2}\sum_{UHP}(\text{residue for } g(z)e^{jB_o d'z})\right.$$
$$\left. + \tfrac{1}{2}\sum_{LHP}(\text{residue for } g(z)e^{-jB_o d'z}, \text{ including } z=0)\right)$$

For I_1, the sum of residues is $\frac{\sqrt{2}}{2B_o d'}$

I_2, $\quad \frac{\sqrt{2}}{4B_o d'} e^{-\frac{B_o d'}{\sqrt{2}}}\left\{3\sin\frac{B_o d'}{\sqrt{2}} + \cos\frac{B_o d'}{\sqrt{2}}\right\}$

residue of $z=0$ is $-j\frac{1}{2\pi}$

I_3, $\quad -1 + \frac{\sqrt{2}}{4B_o d'} e^{-\frac{B_o d'}{\sqrt{2}}}\left\{3\sin\frac{B_o d'}{\sqrt{2}} + \cos\frac{B_o d'}{\sqrt{2}}\right\}$

$$I = \frac{\sqrt{2}}{2B_o d} - \frac{\sqrt{2}}{4B_o d}e^{-\frac{B_o d}{\sqrt{2}}}\left\{3\sin\frac{B_o d}{\sqrt{2}} + \cos\frac{B_o d}{\sqrt{2}}\right\} + 1 - \frac{\sqrt{2}}{4B_o d}\left\{3\sin\frac{B_o d}{\sqrt{2}} + \cos\frac{B_o d}{\sqrt{2}}\right\}$$

$$= 1 + \frac{\sqrt{2}}{2B_o d}\left(1 - 3e^{-\frac{B_o d}{\sqrt{2}}}\sin\frac{B_o d}{\sqrt{2}} - e^{-\frac{B_o d}{\sqrt{2}}}\cos\frac{B_o d}{\sqrt{2}}\right)$$

If B_0d is small enough to neglect second power,

$$I = 1 + \frac{\sqrt{2}}{2B_0d}\left\{1-3\left(1-\frac{B_0d}{\sqrt{2}}\right)\frac{B_0d}{\sqrt{2}} - \left(1-\frac{B_0d}{\sqrt{2}}\right)\right\}$$

$$= \frac{3\sqrt{2}}{4}B_0d = 1.06B_0d$$

Thus $\sigma_{\theta_s}^2 \doteq \theta^2 B_0d$

APPENDIX II

COMPUTER PROGRAM FOR EQUATION (4-18)

```
      PROGRAM ACQUIRE
      DIMENSION X(30),XDOT(30),C(15)
      C(10)=1.
  1   CALL INTEG1(T,X,XDOT,C)
      IF (X(1)+2.0) 10,11,12
 12   IF (X(1)+1.0) 11,13,14
 14   IF (X(1)-1.0) 13,13,15
 15   IF (X(1)-2.0) 16,16,10
 10   DX=0.0
      DXDOT=0.0
      GO TO 2
 11   DX=-2.0-X(1)
      GO TO 3
 16   DX=2.0-X(1)
  3   DXDOT=-1.0
      GO TO 2
 13   DX=X(1)
      DXDOT=1.0
  2   ACCE=C(1)
      SPED=C(1)*T+C(2)
      ERROR=C(3)*SPED+ACCE-(C(3)+1.414213*DXDOT)*X(2)-DX
      XDOT(1)=X(2)
      XDOT(2)=ERROR
      GO TO 1
      END
```

END

For g=infinity

time; from 0. to 9.0, with interval 0.005

C(1)=0.

C(2)=y

C(3)=1/g=0.

Initial value of X(1)=-2.0

X(2)=$\dot{y}$

For g=10

time; from 0. to 14.0, with interval 0.01

C(1)=0

C(2)=y

C(3)=1/g=0.1

Initial value of X(1)=-2.0

X(2)=$\dot{y}$

APPENDIX III

SUBROUTINE INTEG 1

This subroutine provides a simple but moderately flexible means for the solution of ordinary differential equations, employing integration by the fourth-order Runge-Kutta numerical method. Input, and both print and graph output routines are built in. The step size of integration can be changed by appropriate data input and/or programming.

Title: Runge-Kutta Solution of Ordinary Differential Equations
with Built-in Input and Output Routines

CO-OP ID: D2-NPS-INTEG1 (FORTRAN 60)

Programmer: J. R. Ward

Date: June 1964

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The dynamics of phase locked loops have been studied by many investigators for the cases of either a pure sinusodial signal or for pseudo noise (PN) sequence. This dissertation presents an analysis of a phase locked system consisting of sinusoidal carrier modulated with a PN sequence. The effect of the modulation index of the phase shift modulation and the design of a loop filter for coherent detection of a PN sequence are analysed. Additionally, the error probability for each digit, when a likelihood detector is used in front of the delay lock discriminator for a PN sequence, is also converted to the delay error of the phase lock loop.

DD FORM 1 JAN 64 1473

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Phase locked loop						
Delay locked discriminator						
Band limited white Gaussian noise						
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